AN EXPERIMENTAL APPROACH TO THE RHEOLOGICAL CHARACTERIZATION OF BITUMINOUS MIXTURES BASED ON PSEUDO-RANDOM STRESS EXCITATIONS

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ABSTRACT

Rheological behavior of bituminous mixtures in the small strain domain is effectively described by the Linear Visco-Elasticity (LVE) theory. Since the response of LVE materials is equivalent to that of Linear Time-Invariant (LTI) systems, material functions that are used to characterize LVE behavior in the frequency domain can be considered as Frequency Response Functions (FRF) of a LTI system. In this experimental research a pseudo-random stress excitation, comprising a broad band of harmonic components, is used to estimate the LVE response function of a bituminous mixture using signals and systems oriented techniques. Cylindrical specimens were tested in tension-compressions configuration, both excitation (stress) and response (strain) signals were analyzed using a spectral analysis. The FRF estimated using pseudo-random experiments is compared with conventional harmonic analysis results. Advantages and limitations of the proposed approach are highlighted, paying particular attention to the possible effects of non-linearity and viscoplastic deformations.

Keywords: Linear Viscoelasticity; Linear Time-Invariant Systems; Random signals; Frequency Response Function

INTRODUCTION

The rheological behaviour of bituminous materials in the small strain domain is effectively described by the Linear Visco-Elasticity (LVE) theory [1]. This means that the relationship between time-dependent stress ($\sigma(t)$) and strain ($\varepsilon(t)$) can be expressed by the Boltzmann superposition integral [2]. Considering the dualism of the LVE theory, $\sigma(t)$ and $\varepsilon(t)$ can be viewed either as excitations ($x(t)$), or responses ($y(t)$), therefore it is customary to write the superposition integral as:

$$y(t) = \int_0^t h(t-\tau)\, x(\tau)\, \mathrm{d}\tau \qquad (1)$$

where $h(t)$ is a characteristic material function, also called memory or weighting function.

The convolution integral of equation (1) is also known to define the relationship between the input signal $x(t)$, and the output signal $y(t)$, of a continuous-time Linear, Time-Invariant (LTI) and causal system [3] (Cfr. Fig. 1). The material function $h(t)$ is also called the impulse response of the system, in fact it can be obtained as system output when the input is an impulse. The concepts and techniques of signals and systems analysis have been applied to a wide range of real-world phenomena and systems (electrical, mechanical, biological etc.). In this paper LVE materials are viewed as physical LTI systems which transform an input signal $x(t)$, either a stress or a strain, into an output signal $y(t)$.

Figure 1. Representation of the analogy between LVE materials and LTI systems.

The experimental determination of a LVE material function is generally carried out applying a standard excitation and measuring the response. For example, when the input signal is a sinusoidal (harmonic) excitation, either the complex Young's Modulus $E(\mathrm{j}\omega)$ or the complex shear modulus $G(\mathrm{j}\omega)$ are obtained if normal or shear strain are used as input, respectively.

In a system-oriented perspective, the process of obtaining the impulse response of a system starting from the experimentally measured values of the input and output signals is called system identification [4]. When harmonic excitations are employed, the approach is called frequency-response analysis and leads to the determination of a complex-valued, frequency-dependent material function $H(\mathrm{j}\omega)$ called the Frequency Response Function (FRF) of the system. A frequency-response analysis is routinely performed for the rheological characterization of LVE materials: a sinusoidal excitation of fixed frequency (ω) and amplitude (x_0) is applied, the amplitude (y_0) and phase shift (δ) of the steady-state response is measured, and the FRF of the material is directly calculated as:

$$\hat{H}(\mathrm{j}\omega) = \frac{y_0}{x_0} e^{\mathrm{j}\delta} \qquad (2)$$

where j is the imaginary unit. This procedure is repeated for a number of frequencies (frequency-sweep tests) as required by the material properties and possibly limited by instrumental restrictions. If, for example, the input signal is a normal stress ($x(t) = \sigma_0 \exp(\mathrm{j}\omega t)$), and the output signal is a normal strain ($y(t) = \varepsilon_0 \exp(\mathrm{j}\omega t + \delta)$), the mechanical interpretation of equation (2) gives:

$$\left|\hat{H}(\mathrm{j}\omega)\right| = J_0(\mathrm{j}\omega) = \frac{1}{E_0(\mathrm{j}\omega)} \ ; \quad \arg\left(\hat{H}(\mathrm{j}\omega)\right) = \delta(\mathrm{j}\omega) \qquad (3)$$

where $J_0(\mathrm{j}\omega)$ and $E_0(\mathrm{j}\omega)$ are the absolute values of the complex normal compliance and modulus (Young's modulus), while $\delta(\mathrm{j}\omega)$ is the phase angle.

However, since different frequencies pass through LTI systems independently, LVE materials do not need to be excited one frequency at a time [5]. In fact the FRF may be estimated exciting the material with a broadband random signal, composed by a blend of many sinusoids. In this case a raw estimate of the FRF can be directly obtained as:

$$\hat{H}(\mathrm{j}\omega) = \frac{Y(\mathrm{j}\omega)}{X(\mathrm{j}\omega)} \qquad (4)$$

where $X(\mathrm{j}\omega)$ and $Y(\mathrm{j}\omega)$ are the Fourier transforms of the input and output signals, respectively and $\hat{H}(\mathrm{j}\omega)$ is the estimated FRF, also called the Empirical Transfer Function Estimate (ETFE) [4]. The quality of FRF estimate can be improved performing a spectral analysis on the input and output signals. This involves the application of statistical methods for spectral estimation which lead to [4]:

$$\hat{H}(\mathrm{j}\omega) = \frac{\Phi_{xy}(\mathrm{j}\omega)}{\Phi_x(\mathrm{j}\omega)} \qquad (5)$$

where $\Phi_x(\mathrm{j}\omega)$ is the spectrum of the input signal and $\Phi_{xy}(\mathrm{j}\omega)$ is the cross-spectrum of the input and output signals. The spectral analysis summarized by equation (5) is implemented in Matlab high-level functions that allow the calculation of the FRF starting from the sequences of excitation and response signals [6].

In this experimental research the FRF of a bituminous mixture subjected to uniaxial tension-compression tests is determined by means of sinusoidal excitations and pseudo-random excitations. Absolute value and phase of the FRF were obtained with a standard frequency-response analysis (sinusoidal excitations) and with a spectral analysis (pseudo-random excitation). The objective of the study is to demonstrate the applicability of system-oriented techniques to the rheological characterization of bituminous mixtures in the LVE domain and compare the results of the two approaches.

MATERIALS AND METHODS

The Asphalt Concrete investigated in this study is a wearing course mix (AC11 surf) produced in a central asphalt plant, using limestone aggregates and a 70/100 pen binder dosed at 5.3 % by aggregate weight. Two cylindrical specimens were prepared using a gyratory compactor in a 150 mm diameter mould, and then cored to a diameter of 94 mm. The specimen height was 120 mm and air voids content was $V_{m,1}$ = 9.0 % and $V_{m,2}$ = 8.5 %.

A servo-hydraulic press associated with a temperature-controlled chamber was used to perform the mechanical tests. The press is controlled using a specifically developed software which allows the applications of standard waveform excitations (step, sinusoidal, triangular, square) as well as pseudo-random waveforms. The applied axial force was measured using a 20 kN force transducer (HBM U2B), whereas the resulting axial deformation was measured using a couple of strain gauges, glued on opposite sides of each specimen, at mid-height. Conventional bonded wire gauges with polyester resin backing (TML P60) were employed. Gauge length is 60 mm and nominal resistance is 120 Ω.

The experimental program consisted of stress-controlled uniaxial tension/compression test, where both pseudo-random and sinusoidal waveforms were applied to the specimens. Pseudo-random waveforms were constructed choosing a random stress value in a fixed interval ($\pm\sigma_0$) according to the standard uniform distribution (U(0,1)); consecutive stress values were chosen after a fixed time span $T_f = 0.02$ s. The pseudo-random process is described by the following model:

$$\sigma_r(t) = 2 \cdot \sigma_0 \cdot (r_U - 0.5) \sum_{k=0}^{N} \left[u(t - kT_f) - u(t - (k+1)T_f) \right] \tag{6}$$

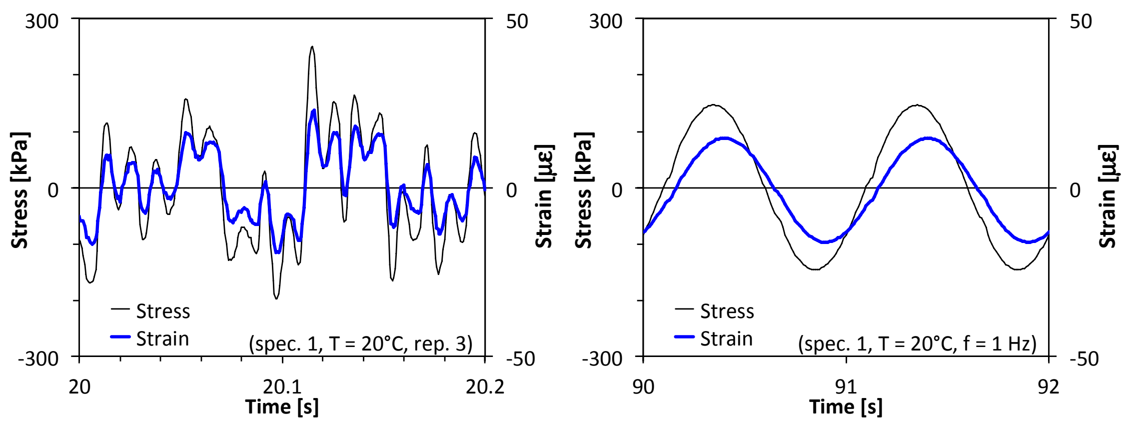
where r_U is a random variable with a standard uniform distribution and $u(t)$ is the unit step function. Tests were carried out at three temperatures (T = 10, 20 and 30°C), three repetitions were performed at each temperature, with a total duration of 120 s for each repetition.

Sinusoidal frequency sweep tests (f = 0.1, 0.25, 1, 4 and 12 Hz) were carried out at five temperatures (T = 0, 10, 20, 30 and 40°C). In each test condition (f and T) the applied sinusoidal stress amplitude was adjusted to obtain a steady state strain amplitude of 15 10^{-6} mm/mm (15 $\mu\varepsilon$).

RESULTS AND ANALYSIS

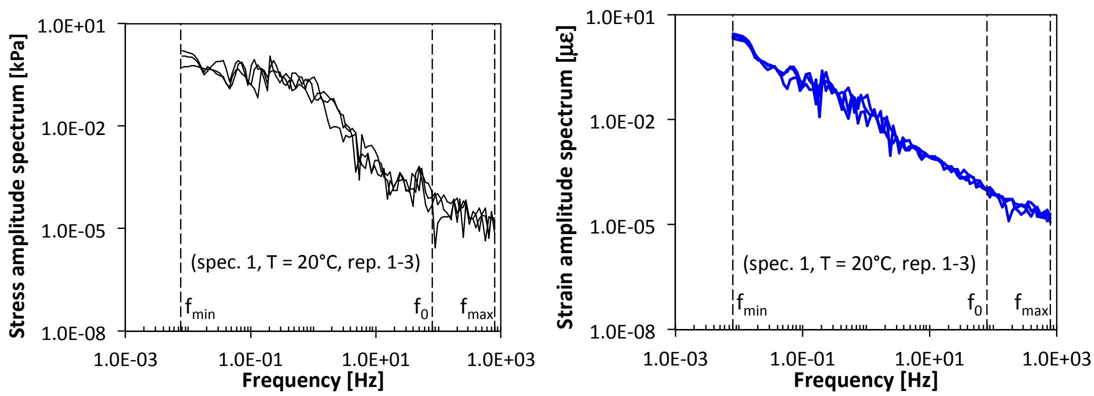
A time domain sample of the pseudo-random and sinusoidal waveform generated by the load actuator are reported in Fig. 2, along with the corresponding measured strain. The random stress waveform (Cfr. Fig. 2, left) is strongly dependent on the frequency response of the electromechanically-operated valve that controls load actuator. For each random test repetition, the standard deviation of the signals may be considered as a representative value of stress and strain amplitude. For example, the random test data showed in Fig. 2 (left), are characterized by standard deviation of axial stress and strain equal to 86.4 kPa and 9.9 microstrain, respectively. In general, for all random repetitions the maximum stress level σ_0 was adjusted to obtain a standard deviation of the strain amplitude below 15 $\mu\varepsilon$ that was the reference strain amplitude of the sinusoidal tests (Fig. 2, right).

Figure 2. Measured pseudo-random (left) and sinusoidal (right) test data



A frequency domain sample of stress and strain random signals (amplitude spectrum) is reported in Fig 3. The minimum and maximum spectral frequencies are the reciprocal of the random test duration ($f_{min} \approx 0.0083$ Hz) and one half the sampling frequency used for data acquisition ($f_{max} = 800$ Hz, the Nyquist frequency), respectively. The spectrum of a white noise signal (random signal containing all frequencies with the same magnitude) should be a constant value. Obviously the actual shape of the stress spectrum produced by the test equipment depends on the pseudo-random process model (Cfr. Eq. 6) and on the electromechanical performance of the load actuator. In particular, frequencies higher than $f_0 = 80$ Hz can be treated as disturbances since the actuator is incapable to generate mechanical vibrations above this frequency (maximum working frequency of the electromechanically-operated valve).

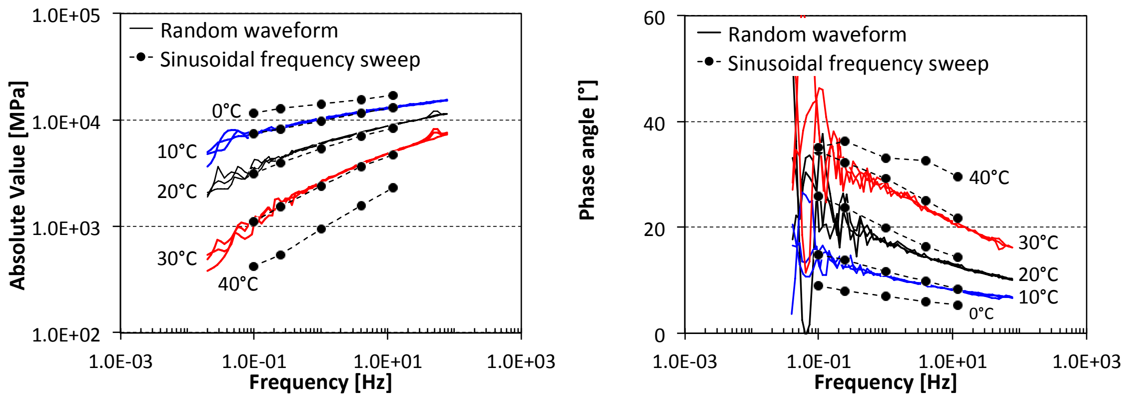
Figure 3. Measured pseudo-random stress (left) and strain (right) amplitude spectrum



For each test repetition, the ETFE of the material was calculated using a spectral analysis (Cfr. Eq. 5) implemented by the Matlab function 'spafdr'. (SPectral Analysis with Frequency Dependent Resolution [6]). In particular, 100 equally spaced points have been chosen on the log-frequency axis. Considering the physical interpretation of the FRF (Cfr. Eq. 3) the ETFE can be plotted in terms of complex Young's modulus absolute value ($E_0(\mathrm{j}\omega)$) and phase ($\delta(\mathrm{j}\omega)$) on a log-frequency scale; this is also called a Bode plot [3].

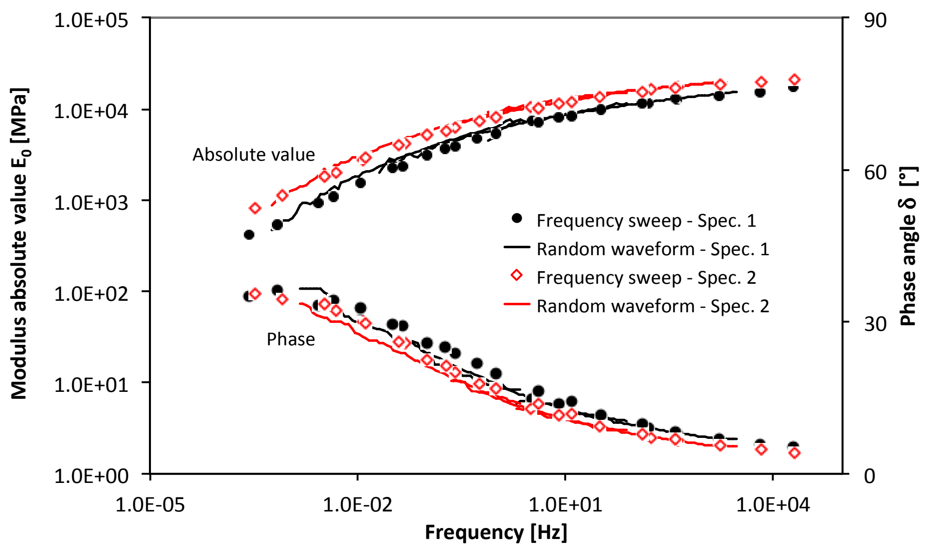
The Bode plot for specimen 1 is reported Fig. 4. Results of the spectral analysis are superimposed to the results of the frequency sweep tests analysed using equation (2) (the classical harmonic analysis). It is highlighted that, with a single test repetition lasting 120 s, complex modulus values at frequencies from 0.02 Hz to 80 Hz were obtained. In addition, the frequency resolution of the results is significantly higher respect to conventional frequency-sweep tests. The $E_0(\mathrm{j}\omega)$ curves obtained from isothermal random repetitions show a variation with frequency and temperature which is consistent with the mechanical behaviour of typical bituminous mixtures. Spectral amplitudes ($E_0(\mathrm{j}\omega)$) and phase values ($\delta(\mathrm{j}\omega)$) are in close agreement with those obtained by means of the conventional frequency sweep approach. Low-frequency noise appears in the $E_0(\mathrm{j}\omega)$ plot and becomes particularly marked in the $\delta(\mathrm{j}\omega)$ plot (Cfr. Fig. 4). This noise may be explained by two mechanical phenomena; first, longer period lengths corresponding to lower spectrum frequencies are comparable with the random test duration and therefore transient effect may be present in the material response and second, at longer excitation periods, non-linearity and viscoplastic effect probably disturb a pure LVE response.

Figure 4. Absolute value $E_0(j\omega)$ (left) and phase $\delta(j\omega)$ (right) of the FRF for specimen 1



The Time-Temperature Superposition Principle (TTSP) is often employed in the rheological characterization of bituminous mixtures to extend the frequency range explored in a single isothermal experiment to a broader frequency range [1]. Fig. 5 shows the application of the TTSP for the construction of Master Curves (reference temperature is 20 °C, both to the frequency-sweep results and to the random tests results. Generally, the frequency response of the random tests is characterized by higher absolute values $E_0(j\omega)$ and lower phase angles $\delta(j\omega)$. This may be explained with material non-linearity, considering that the spectral amplitudes of the random strain response are extremely low (Cfr. Fig. 3, right), if compared with the amplitude of the sinusoidal strain response (Cfr. Fig. 2, right).

Figure 5. Master curves at 20 °C of $E_0(j\omega)$ and $\delta(j\omega)$, for specimen 1 and 2



CONCLUSIONS

In this experimental study the complex Young's modulus of a bituminous mixture was determined adopting an experimental technique based on pseudo-random excitations originally developed for the identification of Linear Time-Invariant systems. A testing equipment, routinely used to perform uniaxial tests on cylindrical specimens, was modified in order to apply pseudo-random stress waves. The upper bandwidth limit of the stress signal (80 Hz) was limited by the electro-mechanical performance of the load actuator whereas its lower limit (0.02 Hz) depended upon the test duration. A spectral analysis was performed in order to obtain the complex Young's modulus in from the measurement of axial strains. Results were in excellent agreement with those obtained from conventional harmonic analysis. In the low-frequency range E* values obtained from the spectral analysis were characterized by an higher noise level which can be related to the presence of a transient material response and to the effect of visco-plastic deformations which disturbed a pure LVE response. The TTSP was applied in order to obtain stiffness and phase angle master curves. In general, the spectral analysis produced slightly higher stiffness and lower phase angle values respect to the harmonic analysis, which can be related to material non-linearity since spectral frequency components are characterized by very small strain amplitudes. The overall results of this study highlight that random waves can be efficiently used to characterize the LVE response of bituminous materials in the frequency domain. Respect to a conventional harmonic analysis the use of random waves allows a substantial reduction of test time and a significant increase of resolution in frequency.

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