

APPLICATION OF LARGE DEFORMATION CIRCULAR CYLINDER TORSION TEST FOR IDENTIFICATION OF MATERIAL MODEL AND PARAMETERS IN CASE OF MODIFIED BINDERS

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ABSTRACT

Constitutive relationships for asphalt binders are usually formulated in the frame of small deformation theory, cf. [1]. However, in many of technological and operational processes during production and laying of asphalt mixes, the binders are subjected to large deformations. Polymer modified binders (eg SBS) in the range of operating temperatures exhibit elastic, viscous and plastic mechanical behaviour. Aim of this study is to describe the elastic properties of modified binders. Due to the large range of deformation the hyperelasticity theory is applied [2-5]. As a result, only processes with monotonic active load are described, at constant temperature and fixed deformation rate. To determine the material functions and parameters for hyperelasticity the results of torsion test (torque and compressive force) carried out in DSR are used, [6,7]. In order to interpret the results of the tests the theoretical solution of torsion boundary problem is applied [4,8].

Keywords: constitutive relationships; hyperelasticity; modified binder; DSR

ANALYTICAL SOLUTION OF TORSION TEST FOR ISOTROPIC INCOMPRESSIBLE HYPERELASTIC MATERIAL

From theoretical point of view the problem of torsion test for large deformation theory is well known in the literature [2-5, 8]. This is an example of a non-uniform deformation, which may be realized in any isotropic incompressible material [2,4,8]. Solution of the torsion of a circular cylinder is useful for programming the

experimental tests for materials showing significant non-linear elastic deformation. The test can be carried out by controlling the resultant force and moment that act on the bases of cylinder subjected to torsion. The constitutive relationship for incompressible isotropic hyperelastic material in Euler description can be written in the following form [2, 4]

$$\boldsymbol{\sigma} = -p\mathbf{I} + \bar{\beta}_1\bar{\mathbf{B}} + \bar{\beta}_{-1}\bar{\mathbf{B}}^{-1}, \quad (1)$$

where

$$\bar{\beta}_1 = 2\frac{\partial W}{\partial \bar{I}_1}, \quad \bar{\beta}_{-1} = -2\frac{\partial W}{\partial \bar{I}_2}, \quad (2)$$

and $W = W(\bar{I}_1, \bar{I}_2)$ is the stored energy function (SEF). Isochoric deformation tensors $\bar{\mathbf{B}}$ and $\bar{\mathbf{B}}^{-1}$ and its invariants \bar{I}_1 and \bar{I}_2 occurring in constitutive equation (1) result from a multiplicative decomposition of the deformation gradient tensor \mathbf{F} into volumetric part $J = \det \mathbf{F}$ and isochoric part $\bar{\mathbf{F}}$, see e.g. [4]. Independent isochoric deformation invariants are defined as follows:

$$\bar{I}_1 = \text{tr} \bar{\mathbf{B}} = \text{tr} \bar{\mathbf{C}}, \quad \bar{I}_2 = \text{tr} \bar{\mathbf{B}}^{-1} = \text{tr} \bar{\mathbf{C}}^{-1}. \quad (3)$$

Tensors $\bar{\mathbf{C}}$ and $\bar{\mathbf{B}}$ are respectively called right and left Cauchy-Green isochoric deformation tensor [2, 4]. In incompressible material the Cauchy stress tensor $\boldsymbol{\sigma}$ is determined without spherical part. Scalar p can be derived by solving of specific boundary-value problem. In torsion test the sample is a circular cylinder of radius a and height h in the initial configuration. Due to the incompressibility restriction and displacement boundary conditions cylinder in the current configuration maintains its dimensions. We assume that the deformation is described by the following relations [4, 8]:

$$r(R, \Phi, Z) = r, \quad \phi(R, \Phi, Z) = \Phi + \alpha Z, \quad z(R, \Phi, Z) = Z. \quad (4)$$

In the cylindrical reference coordinate system each point in the field is determined by three coordinates (R, Φ, Z) , and in the current configuration by (r, ϕ, z) . According to the formulas (4) the height and the radius of cylinder does not change, and the parameter $\alpha = \theta/h$ is the ratio of the rotation angle (unit rotation angle). Therefore, the representation of the deformation gradient tensor, in cylindrical configuration is as follows:

$$\mathbf{F} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{r}{R} & \alpha r \\ 0 & 0 & 1 \end{bmatrix}. \quad (5)$$

Application of the deformation tensor definition $\mathbf{B} = \mathbf{F}\mathbf{F}^T$ gives from (5) the following physical components of tensors \mathbf{B} and \mathbf{B}^{-1} :

$$[B_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 + \alpha^2 r^2 & \alpha r \\ 0 & \alpha r & 1 \end{bmatrix}, \quad [B_{ij}]^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\alpha r \\ 0 & -\alpha r & 1 + \alpha^2 r^2 \end{bmatrix}, \quad i, j = r, \phi, z. \quad (6)$$

The deformation (5) is an isochoric one, that is $J = \det \mathbf{F} = \det \mathbf{B} = 1$ and representations of tensors \mathbf{B} and \mathbf{B}^{-1} according to (6) are respectively the representations of $\bar{\mathbf{B}}$ and $\bar{\mathbf{B}}^{-1}$ tensors. From (6) results also that: $\bar{I}_1 = \bar{I}_2 = 3 + \alpha^2 r^2$. From constitutive relationship (1) and (2) and from (6) it is possible to conclude that physical components of stress true tensor (in local orthonormal basis of cylindrical coordinate system) for any incompressible material are as follows:

$$[\sigma_{ij}] = \begin{bmatrix} \bar{\beta}_1 + \bar{\beta}_{-1} - p & 0 & 0 \\ 0 & \bar{\beta}_1 + \bar{\beta}_{-1} - p + \bar{\beta}_1 \alpha^2 r^2 & (\bar{\beta}_1 - \bar{\beta}_{-1}) \alpha r \\ 0 & (\bar{\beta}_1 - \bar{\beta}_{-1}) \alpha r & \bar{\beta}_1 + \bar{\beta}_{-1} - p + \bar{\beta}_{-1} \alpha^2 r^2 \end{bmatrix}, \quad i, j = r, \phi, z, \quad (7)$$

where material functions depend on r . After application of (7), local balance equations in Euler description: $\text{div} \boldsymbol{\sigma} = \mathbf{0}$ and zero stress boundary condition in current configuration for σ_{rr} on the cylinder peripheral surface, it may be shown that (proceeding as in [4,8]) the determination of the unknown Lagrange multiplier is reduced to the solution of the following ordinary differential equation with variable r :

$$-\bar{\beta}_1 \alpha^2 r + \frac{d\bar{\beta}_1}{dr} + \frac{d\bar{\beta}_{-1}}{dr} - \frac{dp}{dr} = 0, \quad (8)$$

with the initial condition $p(a) = \bar{\beta}_1(a) + \bar{\beta}_{-1}(a)$. In any case, equation (8) with the initial condition can be solved if the material functions (2) are given. The resultant force F and the torque M acting on the cylinder bases, necessary to realize assumed cylinder deformation have the form:

$$F = 2\pi \int_0^a \sigma_{zz} r dr, \quad M = 2\pi \int_0^a \sigma_{\phi z} r^2 dr, \quad (9)$$

which are functions of the unit angle of rotation. The occurrence of the normal force $(10)_1$ in the problem of a circular cylinder torsion theory of hyperelasticity differs substantially from the linear small deformations theory (the presence of so-called Poynting effect). The compression force and torque (10) as a function of the angle of

rotation are measurable quantities in the test. Symbols and operations occurring in the formulas presented in this subsection are classical in mechanics issues, see [2,3,5] and therefore are not explained here.

TORSION TESTS TAKING INTO ACCOUNT NORMAL FORCE

Torsion tests with the axial force measurements were carried out using dynamic shear rheometer HAAKE Mars II, [6,7]. This rheometer has been designed primarily for tests in which the sample in the shape of a circular cylinder is subjected to static or dynamic torsion. In addition, this rheometer allows measuring of the axial force accompanying torsional deformation and realization of tensile deformation with displacement or stress control. DSR is equipped with a thermal chamber with electric heating elements, which works with the cryostat and allows generating the desired temperature program. Tests were carried out on cylindrical samples with a diameter of 8 [mm] and a height of 2 [mm] in the temperature range from 10 [°C] to 50 [°C], using the measurement geometry in the form of two parallel plates. As a result the rheometer realized on sample deformation described above. In these formulas, it is assumed that the sample does not change its height and radius, and that the angle of rotation is linearly distributed on the height of the sample. Torsion of the sample gives a reaction torque and axial force. The tests were intended to be the static one so strain rate was assumed equal to $\dot{\gamma} = 0.07$ [1/s]. The quantity γ is a strain, which in the case of small displacement theory has an interpretation of $\gamma_{\varphi z}(r, \varphi, z)$, and in this case it is precisely the extreme value, i.e. $\gamma = \gamma_{\varphi z}(a, \varphi, z)$. In order to evaluate the statistical properties of the test results of the high modified asphalt binder 65/105-60 performed at a fixed temperature of 20 [°C], the test was carried out on 14 samples, cf. Fig. 1 and 2. Since the experimental results are a functions $M(\gamma)$ and $F(\gamma)$ so for statistical analysis the values of function at fixed (arbitrary chosen) variable γ_i , $i = 1, \dots, 10$ were chosen. For such variable set the estimate of the mean values and standard deviations of the torque $M(\gamma_i)$ and the normal force $F(\gamma_i)$ were evaluated.

Figure 1. Torque as a function of γ in cylinder torsion test of 65/105-60 binder in temperature 20[°C].

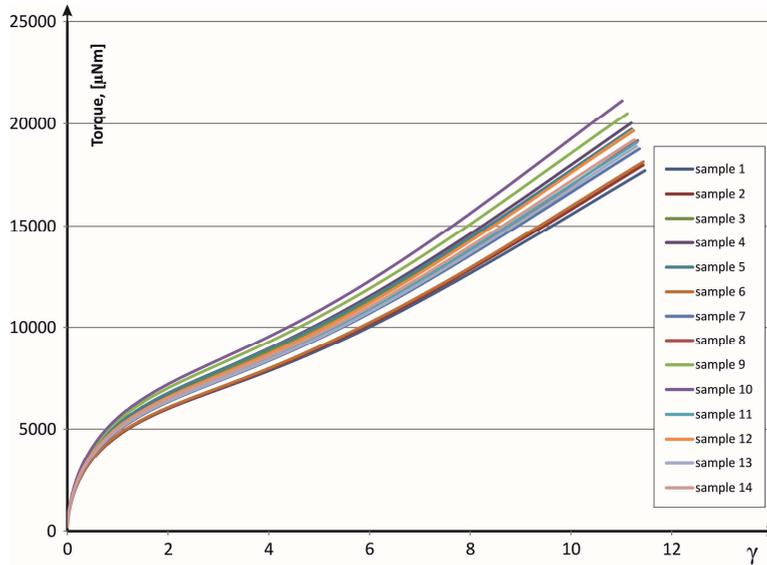
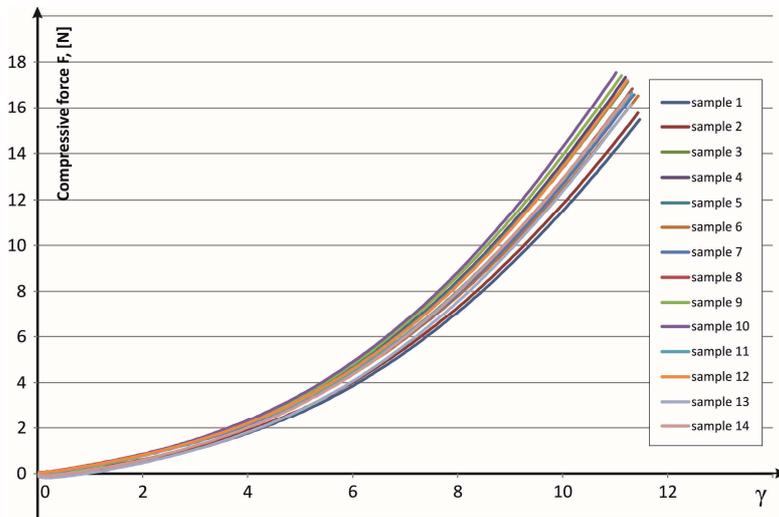


Figure 2. Compressive axial force as a function of γ in cylinder torsion test of 65/105-60 binder in temperature 20[°C].



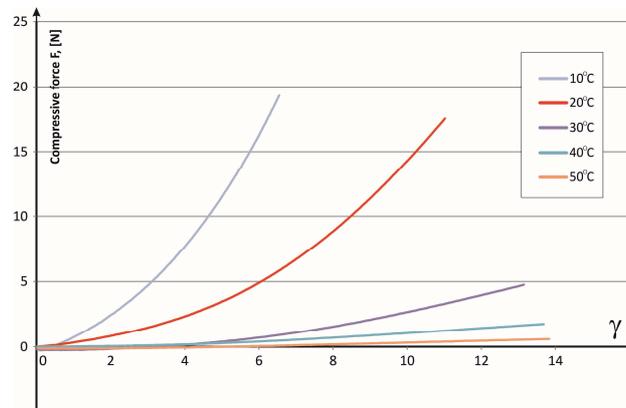
It was assumed that the results of the experiment cannot be separated, i.e. it is impossible to analyze separately the function of the torque and a normal force. It was assumed that according to theoretical analysis it can be concluded that in this experiment, the torque function is the first order effect, and the normal force can be treated as the second order result. Therefore, the decision to reject some of the result can be taken on the basis of the torque function. Accordingly, in the next stage of the statistical analysis the torque results for each variable γ are arranged from smallest to largest, and the absolute value of the difference of the result and the average of the

sample were calculated. In the subsequent step on the basis of Chauvenet criteria it was established that none of the test results should be rejected. Next on the results of the statistical analysis it was assumed that to determine the extent of the confidence interval to a relatively small number of test results the Student's t-distribution will be used. Then, at a confidence level of 95% confidence interval ranges were determined. The results are shown in Tab. 1, where M_s - denotes the mean value of the torque, F_s the mean-value of the normal force, U_M and U_F the confidence intervals for torque and the normal force respectively. Fig. 3 shows the effect of temperature on the results of the normal force in the torsion test with samples made of modified binder. In the case of the normal force with an increase in temperature decreases the absolute value of the force, but this is not a change in an order of magnitude.

Table 1. The results of statistical analysis for values of functions $M(\gamma)$ and $F(\gamma)$ at fixed variable γ_i .

i	γ_i	$M_s [\mu Nm]$	$U_M [\mu Nm]$	$F_s [N]$	$U_F [N]$
1	0.01	470.2	12.5	0.015	0.026
2	0.3	2975.4	80.6	0.030	0.055
3	0.8	4615.8	139.3	0.20	0.065
4	1.35	5646.9	178.5	0.42	0.071
5	1.9	6428.2	223.9	0.71	0.10
6	2.5	7038.0	236.4	1.02	0.08
7	3.7	8254.6	286.3	1.86	0.10
8	5.4	10223.5	363.9	3.62	0.15
9	8.4	14601.3	531.1	8.92	0.33
10	10.3	17684.2	646.0	13.82	0.50

Figure 3. Compressive normal force as a function of the γ in torsion test for binder 65/105-60 - the effect of temperature.



PROPOSITIONS OF CONSTITUTIVE MODELS FOR TORSION TEST DESCRIPTION

In [11] it is shown that there is no theoretical and experimental basis to postulate W as a function independent of \bar{I}_2 invariant. It should be noted, however, that due to the possibility of measurement and the discussion held above, the relationship between torque and rotation angle can be approximated with arbitrary accuracy with the material function resulting from (1) after substituting $\bar{\beta}_{-1}=0$ what means that $W=W(\bar{I}_1)$. On the basis of these considerations, to approximate the results of experimental test, two models with SEF functions dependent from first invariant of isochoric deformation were adopted:

$$W(\bar{I}_1) = \frac{\mu_0}{2} \left[(\bar{I}_1 - 3) + \frac{a_1}{2} (\bar{I}_1 - 3)^2 + \frac{a_2}{3} (\bar{I}_1 - 3)^3 + \frac{a_3}{n_3} (\bar{I}_1 - 3)^{n_3} \right], \quad (10)$$

$$W(\bar{I}_1) = \frac{\mu_0}{2} \left[(\bar{I}_1 - 3) + \frac{a_1}{2} (\bar{I}_1^2 - 9) + \frac{a_2}{3} (\bar{I}_1^3 - 27) + \frac{a_3}{n_3} (\bar{I}_1 - 3)^{n_3} \right]. \quad (11)$$

Fig. 4 shows the two resulting approximations of the experimental tests. Curve (a) corresponds to the SEF function in the form (10), while the curve (b) corresponds to the SEF function in the form (11). The material parameters for the models determined with linear/nonlinear optimization methods are presented in Tab. 2. It is worth noting that the initial values of the shear modulus are based on the interpretation of the solutions in the theory of small displacements. The model predictions were compared with the torque function obtained in experimental tests. In this way, excellent agreement was obtained for torque function, but in the case of compression force function the models prediction are higher than those obtained in measurement, see Fig. 5. However, it appears that the model with SEF function (11) better describes the behavior of the actual material.

Figure 4. Approximations of experimental results in the torsion test of the binder 65/105-60 at $T = 20 [^{\circ}\text{C}]$ using the constitutive model: (a) with SEF (10), (b) with SEF (11).

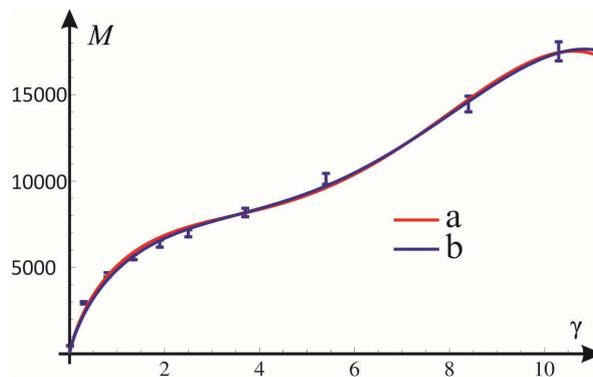


Figure 5. Approximations of experimental results (compressive force) in the torsion test of a cylindrical sample of the binder 65/105-60 at $T = 20 [^{\circ}\text{C}]$ using the constitutive model: (a) with SEF (10), (b) with SEF (11) (material parameters evaluated on the basis of torque function).

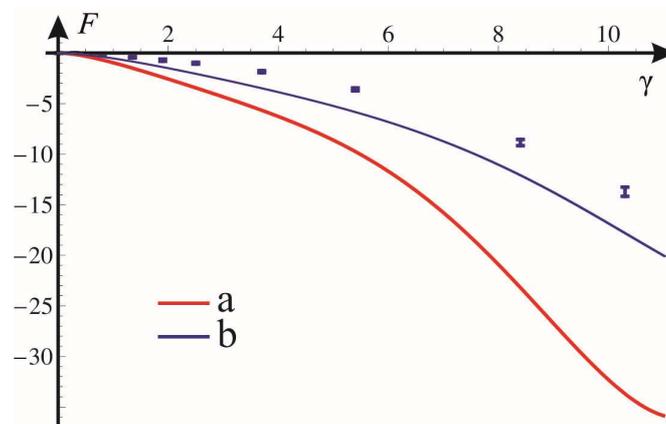


Table 2. Material parameters for selected hyperelasticity models obtained for the binder 65/105-60 (material parameters are based on the function of torque).

	μ_0 [MPa]	a_1	a_2	a_3	n_3
SEF (10)	0.35	0.00105	-4.12E-6	-0.892	1.031
SEF (11)	0.35	0.000927	-3.44E-6	-0.899	1.028

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