EXPLOITATION OF TEST RESULTS TO VALIDATE SOME FORMULAS IN THE REVISED FRENCH RECOMMENDATIONS FOR UHPFRC

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Abstract

Methods or formulas for the design of FRC structures in general, and UHPFRC structures particularly, need validation by tests. In most cases, published FRC structural test results lack a valid characterisation of the material. To remedy this, a preliminary study aims to provide an estimation of the maximum post-cracking strength of fibre concrete, based on a formula including an orientation factor ψ which is the main unknown inducing the great scatter of the experimental results, and the ultimate bond strength of the fibres which can be obtain directly from pull-out tests on single fibres, or indirectly from the experimental post-cracking tensile strength with an assumption on ψ. Then several studies with a validation on test results are done: constitutive law in compression (including confinement effect of fibres); formula for bond strength of rebars in FRC enhanced by fibres; formula to estimate the crack spacing in FRC elements with rebars; exploitation of push-off tests for application to the shear strength of keys in precast elements.

Résumé

L'utilisation d'un matériau nouveau comme le BFUP demande des validations expérimentales tant qu'on n'a pas une expérience suffisante et des modèles fiables. La principale difficulté dans les essais structuraux en BFUP est que généralement le matériau n'est pas caractérisé dans des conditions représentatives de la structure. Une étude préliminaire a validé une formule connue de la résistance maximale post-fissuration $f_{ctf,max}$, qui fait intervenir un facteur d'orientation ψ et l'adhérence ultime des fibres $f_{bf,u}$. En pratique, le facteur ψ est l'inconnue principale qui explique la grosse dispersion des résultats d'essais. L'adhérence $f_{bf,u}$ est obtenue directement d'essais d'arrachement de fibres isolées, ou indirectement de $f_{ctf,max}$ expérimental et d'une hypothèse sur ψ. Ensuite diverses études basées sur les résultats d'essais ont été faites : loi de comportement en compression (effet de confinement des fibres); adhérence des barres ; espacement des fissures des éléments armés et fibrés; résistance au cisaillement des joints à clés entre éléments préfabriqués.

1. INTRODUCTION

The structural use of new materials such as UHPFRC needs validation by tests till a sufficient experience and development of pertinent models are achieved. Experimental validation of some calculation formulas given in the revised French recommendations for UHPFRC [1] was needed. The main difficulty to use test results, for UHPFRC as well as ordinary FRC, is that generally it lacks the characterization of the material in the actual conditions occurring inside the structural element. A preliminary study was done to find an estimation of the maximum post-cracking strength of fibre concrete $f_{ctf,max}$*, based on a well known formula which includes an orientation factor ψ and the ultimate bond strength of the fibres $f_{bf,u}$. Practically, the factor ψ is the main unknown which explains the great scatter of the experimental results. The bond $f_{bf,u}$ can be obtain from test results, with an assumption on ψ. Then several studies based on test results are done: constitutive law in compression (which includes the confinement effect due to fibres); bond strength of rebars in FRC ; estimation of crack spacing in FRC elements with rebars; shear strength of joints keys in precast elements. To increase the number of results, not only tests on UHPFRC were considered but also on ordinary FRC, believing in the continuity of behaviour between these types of concrete.

2. PRELIMINARY STUDY: ESTIMATE THE MAXIMUM POST-CRACKING STRENGTH OF FIBRE CONCRETE

2.1 Formula for the maximum post-cracking strength

The post-cracking maximum strength of fibre concrete $f_{ctf,max}$* is a main parameter involved in the behaviour of UHPFRC structures. When trying to validate calculation formulas on test results of structural elements, in most cases the post-cracking tensile behaviour is not experimentally established by tests representative of the fibre concrete inside the structure. Then it is necessary at first to estimate $f_{ctf,max}$*. This is the maximum of the conventional fibre stress, ratio of the tensile force N_t across the crack to the concrete area A_c : $f_{ctf,max}^* = N_{t,max} / A_c$. We use a simplified reasoning in mean values. The force N_t is transferred to the concrete by bond along the anchorage length of the fibres varying between zero and its maximum $l_f/2$ (mean value $l_f/4$), projected in the direction of N_t by means of the orientation factor ψ. The maximum force $N_{t,max}$ and strength $f_{ctf,max}$* are reached when all the fibres are at their ultimate bond strength $f_{bf,u}$ (fibre length l_f, diameter d_f, possibly an equivalent diameter) :

$N_{t,max} = A_c f_{ctf,max}^* = \psi\, n_f\, \pi\, d_f\, (l_f/4)\, f_{bf,u}$.

The number of fibres across the crack is $n_f = V_f A_c / (\pi d_f^2 / 4)$. Then the well known formula:

$$f_{ctf,max}^* = \psi\, V_f\, (l_f/d_f)\, f_{bf,u} \qquad (1)$$

2.2 Orientation factor

The mean orientation factor ψ has a purely geometric definition which results from the projection on the direction of N_t as a function of the individual angle α_i of the fibre with this direction. If fibres are all parallel to the same direction: $\alpha = 0$, $\psi = 1$; $\alpha = \pi/2$, $\psi = 0$. For a 2D isotropic distribution: $\psi = (2/\pi) \int_0^{\pi/2} \cos\alpha \, d\alpha = 2/\pi = 0.637$. For a 3D isotropic distribution: $\psi = (2/\pi)^2 = 0.405$. Practically the extreme values 0 and 1 will never be reached in a large volume of fibre concrete, and roughly $0.2 < \psi < 0.8$. Really, ψ is a mean efficiency factor, resulting mainly from the orientation (only considered here) but also from the fact that the fibres with small angle α_i must be kinked across the crack to increase α_i before reaching their ultimate bond. For appropriate shape and dimension of formwork, and with a well defined procedure of casting, a 3D isotropic distribution may be expected in the current volume, as supposed in the French recommendations [1] for the prismatic control specimens. But the distribution is locally modified by the wall effect of the formwork and by sawn planes (see fig. 1 and [1] appendix 2; longitudinal effect only). When reliable information exist concerning the real orientation, the current value ψ = 0.40 may be modified. In this study, these effects were generalized for cylindrical specimens and for transverse direction and transverse walls. In fig. 1, under the assumptions of 3D isotropic distribution ψ = 0.40 in the current volume (area A3) and parabolic variation between the walls and the limit of A3, or between the angle and the limit of A2, for the longitudinal x direction the mean factor of orientation is taken as 0.68 in area A1, 0.48 in A2 and 0.20 in A4 (area limited by a sawn plane).

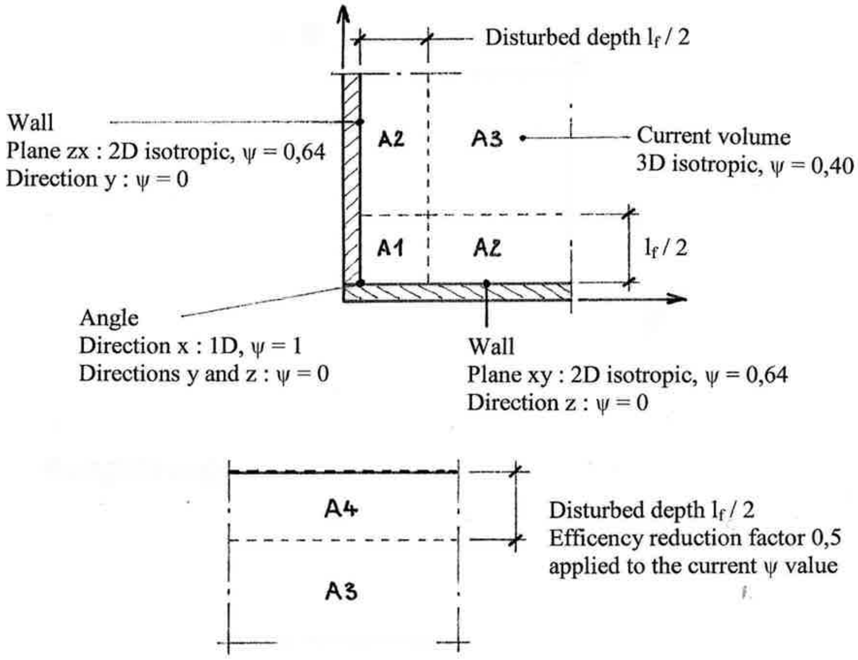

Figure 1: Orientation factor ψ. 1) Wall effect of formwork. 2) Effect of sawn plane.

2.3 Ultimate bond strength of fibres

The ultimate bond strength can be expressed by:

$$f_{bfu} = \alpha\, \eta_f\, f_{ctf} \quad (2)$$

where α is a numerical factor, η_f a shape factor depending on the type of fibre and f_{ctf} the tensile strength (first cracking) of the fibre concrete matrix. The values of η_f in table 1 are in the same range as the values η_s and η_p for for rebars and prestressing bars.

Table 1: Shape factor η_f

- Plain straight	- Hooked - Crimped	- Twisted - Paddled ends	- Straight indented	- Conical ends - Bi-undulated	- Trombone
η_f = 0,8	1,3	1,5	1,7	2,0	2,2

The experimental support deals with tests not only on UHPFRC but also on ordinary FRC. Pull-out tests on isolated fibre [2] to [6] give directly the factor α for a known value of η_f, but they are too few. Therefore α is also indirectly estimated from the strength $f_{ctf,max}$* measured in direct tension tests [6] to [17], whence f_{bfu} by equation (1), then α. The main uncertainty results from the misappreciation of the orientation factor ψ as was already mentioned. From the analysis of 33 tests, α = 1.65 is retained. The table 2 shows that this value is practically the same directly from pull-out tests or indirectly from tension tests, the same for hooked fibres and straight fibres, and the same for UHPFRC and for ordinary FRC.

The tensile strength f_{ctf} is not always well known. So a correlation was also searched for with the compressive strength f_{cf}, assuming that f_{ctf} is proportional to $f_{cf}^{2/3}$:

$$f_{bf,u} = \beta\, \eta_f\, f_{cf}^{2/3} \qquad (3)$$

Results are shown in table 2, with a little more scatter than α, resulting in β = 0.42.

Table 2: Values of α and β obtained from tests.

	Number of results	α mean	Number of results	β mean
Fiber pull-out tests (direct f_{bfu})	7	1.65	4	0.415
Direct tension tests (indirect f_{bfu})	26	1.66	23	0.42
Hooked fibres	18	1.66	16	0.415
Plain straight fibres	9	1.69	6	0.445
UHPFRC (with straight fibres)	11	1.70	8	0.44
Whole results	**33**	**1.66**	**27**	**0.42**

3. CONSTITUTIVE LAW IN COMPRESSION. CONFINING PRESSURE DUE TO FIBRES

3.1 Empirical constitutive law

An empirical model valid for the complete stress-strain law of concrete without fibres [18], which was validated by a large number of tests for ordinary concrete and for HPC, has been extrapolated to fibre concrete with a large range of strength, from ordinary FRC to UHPFRC. The model is based on 4 parameters: tangent modulus E_{c0}, strength f_c, peak strain ε_{c1}, strain ε_{c2} at a particular point at 0.7 f_c on the falling branch. Assuming E_{c0} and f_c be known, the factor k_0 is known in the following equation:

$$E_{c0} = k_0\, f_c^{1/3} \qquad (4)$$

The two other parameters are then calculated from:

$$\varepsilon_{c1} = [1 + 0{,}16\, k_0 / (f_c^2 + 800)]\, (f_c^{2/3} / k_0) \qquad f_c \text{ in MPa} \qquad (5)$$

$$\varepsilon_{c2} = (1 + 20 / f_c)\, \varepsilon_{c1} \qquad (6)$$

The extrapolation is based on the strength f_{cf} and tangent modulus E_{c0f} of the fibre concrete still related by (4). About 50 experimental records of the stress-strain curve, from 13 authors or groups of authors [9] [16] [19] to [32] have been analysed; in some cases, a result is the mean on several tests. There are less results for ε_{c2} than ε_{c1}. The following formulas are proposed for the strains in fibre concrete $\varepsilon_{c1,f}$ and $\varepsilon_{c2,f}$, related to the maximum post-cracking strength $f_{ctf,max}$*:

$\varepsilon_{c1,f} = (1 + \ \ 4\ f_{ctf,max}^* / f_{cf})\ \varepsilon_{c1}$ (7)

$\varepsilon_{c2,f} = (1 + 15\ f_{ctf,max}^* / f_{cf})\ \varepsilon_{c2}$ (8)

Here, for ε_{c1} and ε_{c2}, f_c is replaced by f_{cf} in (5) and (6). In $f_{ctf,max}^*$, the orientation factor ψ is that in the radial direction (confining effect).

3.2 Confinement effect due to fibres

Fibres give a confinement effect equivalent to a radial pressure σ_{2f}. A theoretical analysis by analogy with orthogonal hair-pin confining reinforcement, shows that σ_{2f} might be in the range $0.5 < \sigma_{2f} / f_{ctf,max}^* < 2$. A relation is searched for:

$\sigma_{2f} = \gamma\ f_{ctf,max}^*$ (9)

Formula (7) of $\varepsilon_{c1,f}$ is compared to that of MC CEB-FIP 1990 [33] for $\varepsilon_{c1,c}$ confined by σ_2 :

$\varepsilon_{c1,c} = (f_{c,c} / f_c)^2\ \varepsilon_{c1}$ ($f_{c,c}$ confined strength ; ε_{c1} for unconfined f_c)) (10)

$f_{c,c} = (1 + 5\ \sigma_2 / f_c)\ f_c$ (11)

In (7) the confinement effect on the strength is already included in f_{cf} and ε_{c1} is referred to $f_{c,c}$. So it may be assumed to use power 1 in lieu of 2 in (10):

$\varepsilon_{c1,c} \approx (1 + 5\ \sigma_2 / f_{c,c})\ \varepsilon_{c1}(f_{c,c})$ (ε_{c1} for confined $f_{c,c}$) (12)

Compared to (7) it gives $\gamma = 0.8$. The confinement effect in $\varepsilon_{c2,f}$ is more difficult to analyse, but is surely less than in $\varepsilon_{c1,f}$. It seems that $\gamma > 0.2$ for ordinary FRC and $\gamma > 0.3$ for UHPFRC. For ordinary FRC, flexural and shear tests on beams with circular cross section [34] show that fibres can replaced partly the hoop transverse reinforcement, with an equivalence $\gamma \approx 0.55$.

In conclusion, the following values may be considered but need further validation: in elements mainly subjected to compression $\gamma \approx 0.8$ and to flexure $\gamma \approx 0.5$.

4. BOND STRENGTH OF REBARS IN FIBRE CONCRETE

The interim French recommendations for UHPFRC [35] were mainly oriented towards prestressed structures in which the use of UHPFRC generally exempts to put ordinary reinforcement. The revised recommendations [1] aim to enlarge their field of application and to cover all combinations of rebars and prestressing bars with UHPFRC, in the format of the Eurocode 2 EN 1992-1-1 [36]. An item was the enhancement of the rebars bond by fibres.

Prejudging the large scatter to be find in the tests results, only the simplest correlation of rebars bond $f_{bsu,f}$ with the maximum post-cracking strength $f_{ctf,max}^*$ was search for :

$f_{bsu,f} = \delta\ f_{bsu}$; $\delta = f_{bsu,f}$ with fibres / f_{bsu} without fibres $= 1 + \theta\ (f_{ctf,max}^* / f_{tf})$ (13)

A little more than 60 experimental results are considered, in 17 series (generally each result is a mean on several tests), from references [6] [37] to [44]. In pull-out test the bar goes through the whole length of the specimen; in the estimation of the orientation factor ψ, all the concrete volume is concerned and it is necessary to take in account the effect of the transverse walls of the formwork. Furthermore, the value of $f_{ctf,max}^*$ to be considered is the transverse circumferential one, which opposes to the splitting of the concrete cover to the rebar (different from the confinement effect in compression, related to the transverse radial value of $f_{ctf,max}^*$). There are two different cases for the evaluation of δ : «direct» when there are tests with and without fibres in the same series, «indirect» when there is no test without fibres. In this second case, the ultimate bond strength f_{bsu} without fibre is calculated using the Model Code CEB-FIP formula [33], the same more or less considered by EC 2 [36], without safety factors.

As usually considered, the ultimate bond strength f_{bsu} is the mean value on the entire embedded length l_c. To be consistent with the development of bond along the bar assumed in the calculation of crack spacing (see § 5), a length ℓ_0 without bond should be considered. It was not used here, but only the tests where the embedded length l_c is sufficiently high are considered, say $l_c >$ about 2/3 l_{su}, where l_{su} is the ultimate anchor length (without safety factor); too small values of l_c («local» bond tests) are rejected. The mean value for all the 17 series of tests is $\theta \approx 0.6$. Direct evaluation on 8 series results in $\theta \approx 0.45$; indirect on 9 series, $\theta \approx 0.75$. Effect of the relative cover to bar diameter c / Ø: it is quite clear that the enhancement due to fibres is greater when the cover c is small, i.e. when the failure mechanism is splitting. Except 2 series where c / Ø varies largely with a rather aberrant effect: $c \leq 3$ Ø (6 series) $\theta \approx 0.8$ (direct evaluation, 2 series, $\theta \approx 0.7$) ; $c > 3$ Ø (9 series) $\theta \approx 0.55$ (direct, 6 series, $\theta \approx 0.4$).

For UHPFRC, the enhancement would be rather higher. Mean value on 8 series: $\theta \approx 0.65$ (direct evaluation, 2 series, $\theta \approx 0.6$). Cover effect: $c \leq 3$ Ø (2 series), $\theta \approx 1.05$; $c > 3$ Ø (4 series), $\theta \approx 0.65$. An acceptable value $\theta = 0.5$ was retained for the revised recommendations [1], so $\delta = 1 + 0.5\ (f_{ctf,max}^* / f_{ctf})$, which is probably too safe for UHPFRC and surely an underestimation when the relative cover is small ($c < 3$ Ø).

5. ESTIMATION OF CRACK SPACING IN FRC ELEMENTS WITH REBARS

The formula for crack spacing is primarily that of Model Code CEB-FIP 1978 [45], slightly modified in Eurocode 2 [36]. The minimum distance from a first crack at which a second one can formed is $s_{r,min} = \ell_0 + \ell_t$. The first term ℓ_0 roughly speaking express that there is no bond in the vicinity of the first crack because very high tensile stresses and micro-cracking are induced in a limited area of concrete around the bar ; Model Code and EC 2 takes it as 1,33 times the cover c. The second term ℓ_t is the length necessary to transfer by bond from bar to concrete a tensile stress σ_{ct} reaching the tensile strength f_{ct}: π Ø $\ell_t f_{bsu} = A_c$ times f_{ct}, with $f_{bsu} \approx 1.67\ \eta_s f_{ct}$ (η_s shape factor). Replacing A_c / π Ø by Ø / 4 ρ (rebar ratio ρ) the transfer length is then:

$$\ell_t = (0.15 / \eta_s)\ (Ø / \rho) \qquad (14)$$

After cracking, $\sigma_{ct} = 0$. The rebar ratio is ρ_{eff} related to the efficient area $A_{c,eff}$ of the tie ; according to a new definition of $A_{c,eff}$ in Eurocode 2, the length ℓ_t is multiplied by a factor 2. It is also multiplied by k_2 to differentiate pure tension ($k_2 = 1$) and flexure ($k_2 = 0.5$).

For FRC, the calculation of crack spacing is only pertinent if the post-cracking behaviour is softening. In [1] three adaptations are made for FRC. 1) Enhancement of the concrete strength along ℓ_0 by fibres: the first term is divided by δ_1, which is taken equal to δ as a first approach. 2) Enhancement of bond of rebar : $f_{bsu,f} = \delta\ f_{bsu}$, so the second term is divided by δ. 3) It is also limited to the minimum spacing of cracks in hardening FRC supposed to be l_f / 2. After cracking, $\sigma_{ct} = f_{ctf,max}^*$, so hereunder in (15) f_{ct} is replaced by $f_{ctf} - f_{ctf,max}^*$. The mean crack spacing, which is to be compared to test results, is taken as $s_{rm} = 1.5\ s_{r,min} = 1.5\ (\ell_0 + \ell_t)$. For FRC the mean spacing writes:

$$s_{rm,f} = 2\ c / \delta + [2 \times 0.225\ k_2\ (1 - f_{ctf,max}^* / f_{ctf}) / \delta\ \eta_s]\ (Ø / \rho_{eff}) \geq 0.75\ l_f \qquad (15)$$

The number of available test results is not great, 7 from tie members [6], [46] to [48], and 13 from beams [49] to [53]. Despite the great scatter, the formula may be considered as validated: mean ratio test / calculation ≈ 1.05 (0.6 to 1.3, mean 0.85 for tie members; 0.5 to 1.8, mean 1.15 for beams).

6. SHEAR STRENGTH OF JOINTS KEYS IN PRECAST ELEMENTS

6.1 Cohesion and friction

The object of this part is to adapt to UHPFRC the § 6.2.5 *Shear along construction joints* of Eurocode 2 [36] and specially the formula (6.25) which gives the resisting shear strength v_R, comprising three terms, a concrete cohesion term v_{Rc}, a friction term due to reinforcement passing through the shear plane v_{Rs} and a friction term due to external compression σ_N. Without safety factors (for comparison to tests) it is written:

$$v_R = c\ f_{ct} + \rho\ f_y\ (tg\varphi\ \sin\alpha + \cos\alpha) + tg\varphi\ \sigma_N \quad \leq\ v_{R,lim} \qquad (16)$$

$$v_{R,lim} = 0{,}3\ (1 - f_c/250)\ f_c \qquad (17)$$

For UHPFRC the limit value was modified according to French rules for HPC:

$$v_{R,lim} = 1{,}14\ \alpha_{cc}\ fc^{2/3} \quad \text{with } \alpha_{cc} = 1 \text{ or } 0{,}85 \text{ for loads of short or long duration.} \qquad (18)$$

The cohesion factor c and friction coefficient tgφ depend on the roughness of the construction joint or on the indentations (shear keys) of the joint between precast elements. In the EC 2 provisional version ENV 1992-1-1 [54], it was extended to monolithic concrete.

For UHPFRC, the rules are not modified for construction joints; it is only a question of appreciation of the roughness as a function of concrete fluidity. The coefficient tgφ is that of contact friction between the surfaces. In the case of indentations (and monolithic behaviour), one considers an internal shear failure of concrete; c f_{ct} is an internal cohesion and tgφ an internal friction coefficient, which depend on the strengths f_c and f_{ct} of concrete. In EC 2, within the range 20 ≤ fc ≤ 98 MPa: c = 0.35 constant (but applied to f_{ct} which increases with f_c) and tgφ = 1.35 constant (monolithic from ENV: c = 0.625, tgφ = 1.50). For UHPFRC, c is retained but tgφ must be re-evaluated. For ordinary concrete [55], tg φ can be related to f_c by $tg\varphi \approx 0{,}5\ f_c^{1/3}$ (f_c in MPa). This expression was modified to:

$$tg\varphi \approx 0{,}375\ f_c^{1/3} \qquad (f_c \text{ in MPa}) \qquad (19)$$

so as to find again the EC 2 value for indentations at $f_c \approx 47$ MPa and to have a rather safe extrapolation to UHPFRC, which gives a mean value tgφ ≈ 2.1 in the range $150 < f_{cf} < 250$ MPa (monolithic tgφ ≈ 2,3).

6.2 Contribution of fibres

If the dimensions of the keys are sufficiently high with respect to the fibres length l_f (see [1]), a term of contribution of fibres v_{Rf} is added, evaluated by analogy with the term v_{Rs} due to rebars (without safety factors):

$$v_{Rs} = \rho\ f_y\ (tg\varphi\ \sin\alpha + \cos\alpha) \quad ; \quad \pi/4 \leq \alpha \text{ (angle with shear plane)} \leq \pi/2 \qquad (20)$$

It comprises a term due to friction and a term of direct resistance parallel to the shear plane ; only rebars inside a defined range of inclination α are supposed to be efficient. By analogy, only the fibres in this range are considered, with the mean projection factor ψ associated with this range, thence:

$$v_{Rf} = (0{,}35\ tg\varphi + 0{,}3)\ f_{ctf,max}^* \qquad (21)$$

Indentations: $v_{Rf} = 0.77\ f_{ctf,max}^*$ (ordinary concrete) or $1{,}03\ f_{ctf,max}^*$ (UHPFRC). Monolithic concrete: 0.825 or 1.11.

6.3 Experimental validation

It uses primarily the results of monolithic push-off tests (fig. 2), where a mean stress $\tau = F / H L$ is induced in the shear plane by the compressive force F. The difficulty to interpret such tests lies in the influence on the cohesion term v_{Rc} of a parameter not taken into account by formula (17), the normal stress σ' parallel to the shear plane. Here σ' is a compression. With the same type of specimen, pull-out test where F is a tensile force inducing a tensile stress σ' gives a strength v_{Rc} lower than push-off test. Happily, generally there are both tests with fibres ($v_R = v_{Rc} + v_{Rf}$) and without fibres (v_{Rc}), so as it is possible to isolate v_{Rf} by subtracting v_{Rc}. Estimation of the orientation factor ψ in the direction perpendicular to the shear plane must take into account the effect of the notches, especially when they are sawn. Combined with short length L with respect to l_f, ψ may be as low as 0.28.

About 40 test results are exploited [56] to [60]. The ratios experimental v_{Rf} /calculated value lie between 0.8 and 2.0 with a mean value of 1.45 ; i.e. mean $v_{Rf} \approx 1.2\ f_{ctf,max}$* (between 0.7 and 1.7) to be compared to 0.825 $f_{ctf,max}$* (22). The formula is then validated, with a margin of safety, for ordinary FRC (f_c = 29 to 80 MPa). Unhappily, there are no results for UHPFRC ; the resisting mechanism being the same the formula can be extrapolated in principle, but the value of tgφ for UHP concrete still needs to be validated.

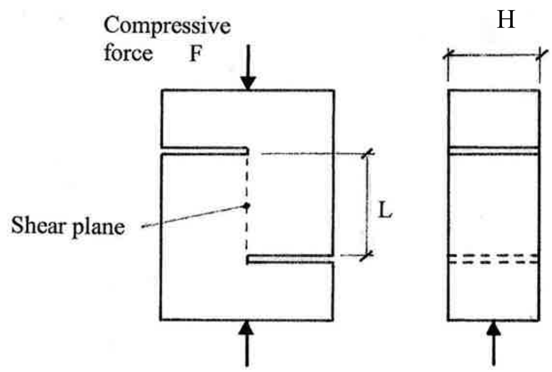

Figure 2: Push-off test.

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