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EXPLICIT BACK ANALYSIS METHOD FOR QUICK DETERMINATION OF DIRECT TENSILE STRENGTH OF PLATE STRUCTURAL MEMBERS

Gilles Chanvillard (1) and Dominique Corvez (2)

(1) Lafarge Research Center, St Quentin Fallavier, France

(2) Lafarge Ductal, Paris, France

Abstract

It is now common to characterize the tensile behaviour of fibre-reinforced concrete by bending tests rather than by direct tensile tests. Then it is fairly common today to apply reverse analysis to extract the tensile constitutive equation of the material. For the case of thin plates showing a bending hardening behaviour, a new explicit method (which does not need any numerical solver) allowing performing reverse analysis into equivalent strain is proposed which starts with an experimental moment-deflection relationship to get tensile stress-strain constitutive relationship. The great advantage of this explicit approach is its easy programming and it allows to quickly manually adjusting a constitutive material relationship on the experimental results without any data pre-treatment. Two specific application cases are presented showing how this methodology could accelerate the use of UHPFRC in real applications making easy the analysis of characterization tests.

Résumé

Il est désormais courant pour caractériser le comportement en traction des bétons renforcés de fibres par des essais de flexion plutôt que par des essais de traction directe. La mise en œuvre d'analyse inverse permet ensuite d'extraire la loi de comportement en traction du matériau. Dans le cas de plaques minces ayant un comportement en flexion écrouissant, une nouvelle méthode explicite (qui ne nécessite pas de solveur numérique) est proposée qui permet de conduire cette analyse inverse en considérant une déformation équivalente en traction. Le caractère explicite facilite la programmation informatique et permet à l'utilisateur d'ajuster manuellement et visuellement la loi de comportement recherchée sans aucun pré-traitement des données expérimentales. Deux cas d'application sont présentés montrant comment cette méthodologie permet d'analyser rapidement des essais de caractérisation et ainsi de faciliter l'utilisation des BFUP dans des applications réelles.

1. INTRODUCTION

The active development of fibre-reinforced concretes over the last 30 years has shown that these materials can be broken down into many mix design making it necessary to develop appropriate methods to characterize their performance.

Around the world, working groups develop recommendations for their use depending on mechanical potential. As fibres contribute primarily in the process of crack control, the first segmentation is carried out on the direct tensile behaviour of the material [1]. Two cases are generally distinguished depending on the hardening/softening behaviour in direct tension. For hardening materials, the tensile constitutive equation could be described as a stress-strain relationship up to the maximum post-cracking stress. For softening materials in tension, in the sense that the post-elastic maximum stress does not exceed the matrix cracking strength, it becomes tricky to define "plastic" strain as cracks would localize depending on element geometry. Nevertheless, the flexural behaviour may present a hardening behaviour. It suffices that the post-cracking strength immediately brought by the fibres after cracking exceeds 50% of the matrix cracking strength [2].

So, what about UHPFRC? Two main properties are essential for fibre-reinforced concrete at structural scale: First crack strength of the matrix and post-cracking behaviour provided by the fibres. Everyone knows that the higher the first crack strength, the higher should be the fibre content (for a given type of fibre) to ensure hardening tensile behaviour. This becomes a challenge if we consider also the rheology and homogeneity. Accordingly, when the strength of the matrix approaches 10 MPa, obtaining a real hardening material in direct tension still possible but could be achieved only with high fibre content which economic justification is often difficult.

However, UHPFRC with high matrix strength presents a great potential for thin elements (length over thickness ratio higher than 50 and thickness limited to three times the length of the fibres). Indeed, a strong multi-cracking is then obtained. It is then accepted to describe the tensile constitutive equation law as a stress-strain relationship up to the maximum post-cracking bending moment. It then generates the need for inverse method to extract this tensile constitutive equation from flexural tests on thin elements. This is the objective of this paper, to develop an easy to use but reliable inverse method for thin elements.

2. BACK ANALYSIS METHOD SPECIFICATIONS

A rigorous, robust, reliable and fast inverse method for thin elements is expecting. It should be recognized that past ten years, many reverse methods have been proposed [3-9]. Remaining with stress-strain relationship, crack opening is no longer required and make it much simpler applying mechanical equilibrium equations. Then the next question is how many points to describe the post-cracking behaviour? Too few may introduce bias as generally the assumption of linear behaviour between the characteristic points is done. As much as possible (the Holy Grail of a scientific point of view) would need an automatic resolution of equations and thus the use of numerical solvers. Practically, the main difficulty comes directly from the quality of experimental data which are necessarily noisy and create strong numerical instabilities. Robustness and reliability is then questionable, and also the user-friendliness.

Based on this analysis, the specifications are the following for the development of a new method:

- The constitutive equation should be described by a sufficient number of points to capture the nonlinearities of the material behaviour. The features of interest are the tensile strength at the first crack and the elastic modulus of the matrix, the strength provided by the fibres after the first crack, the ultimate deformation corresponding to the maximum moment.
- The mechanical equilibrium calculations will be performed by conventional numerical integration methods including in the elastic range. This requires the precise measurement of the deflection to be performed using a device attached to the sample. This could be easily checked by the perfect linearity of the load-deflection experimental curve in the elastic range.
- It falls to the user to adjust the constitutive equation. This implies two specifications. On the one hand, the calculations must be instantaneous while remaining reliable. On the other hand, a simple visual superposition of the calculated curve with the experimental curve allows making the adjustment removing de facto experimental noises. The concept of inverse analysis is therefore to give the hand to the user and simply to make live calculations from a dynamically adjust constitutive equation.

3. MECHANICALLY EXPLICIT MODEL FOR THIN PLATES IN FLEXURE

The behaviour of a section subject to bending taking into account the nonlinear character of the tensile constitutive equation of the material, expressed as a discrete stress-strain relationship is described in this section. At first, an explicit method is proposed to generate the moment-curvature relationship. Then from the moment distribution along the specimen, the deflection could be computed taking into account the non-linear aspect of the moment curvature relationship. Finally, some numerical tricks will be presented that make the application of the model faster and more robust.

3.1 Mechanical Analysis of a section

Figure 1 defines the strains ($\varepsilon=\chi.z$) where χ is the curvature and stresses distributions in the rectangular section (height h, width b) with the main notations used thereafter. Index "i" defined the iteration step, the loading being controlled incrementally on the tensile strain of the bottom fibre (ε_i). The behaviour of the uncracked material is assumed elastic with a modulus of elasticity E. Mechanical equilibrium for the section returns to ensure the balance of the normal force and bending moment.

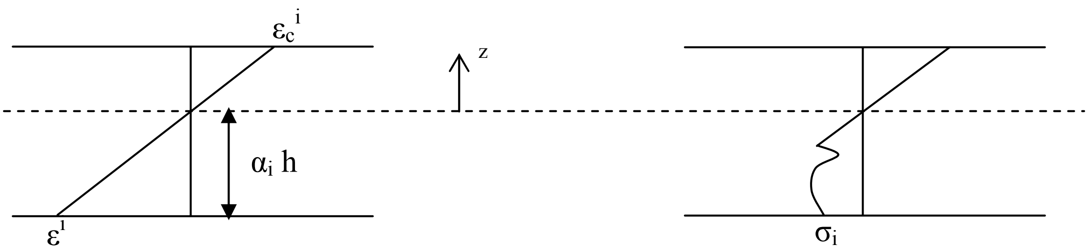

Figure 1: Strain and stress distribution over the height of a section

Normal force is obtained by integrating the stress distribution over the section:

$$N_i=\int_S \sigma.b.dz=\frac{b}{\chi_i}\int_{\varepsilon^i}^{\varepsilon_c^i}\sigma(\varepsilon).d\varepsilon \quad \text{with } dz=\frac{d\varepsilon}{\chi_i} \tag{1}$$

In pure bending, the normal force is zero and compression and tension terms could be separated:

$$\int_{\varepsilon^i}^{0} \sigma(\varepsilon).d\varepsilon + \int_{0}^{\varepsilon_c^i} \sigma(\varepsilon).d\varepsilon = 0 \qquad (2)$$

Assuming a linear strain distribution, a simple geometrical relation links maximum tensile and compressive strain the deformations (figure 1):

$$\varepsilon_c^i.\alpha_i = -\varepsilon^i\left(1-\alpha_i\right) \qquad (3)$$

Using (2) and (3), and considering linear elasticity in compression, on gets:

$$\int_{0}^{\varepsilon^i} \sigma(\varepsilon).d\varepsilon = \frac{E}{2}\left(\varepsilon^i\right)^2\left(\frac{1-\alpha_i}{\alpha_i}\right)^2 \qquad (4)$$

For the next loading stage, index “i+1”, an incremental approach is adopted:

$$\int_{0}^{\varepsilon^{i+1}} \sigma(\varepsilon).d\varepsilon = \int_{0}^{\varepsilon^i} \sigma(\varepsilon).d\varepsilon + \int_{\varepsilon^i}^{\varepsilon^{i+1}} \sigma(\varepsilon).d\varepsilon \qquad (5)$$

First term of (5) is given by (4). The second is approximated using the trapezoidal rule:

$$\int_{\varepsilon^i}^{\varepsilon^{i+1}} \sigma(\varepsilon).d\varepsilon = \frac{1}{2}\left[\sigma(\varepsilon^{i+1})+\sigma(\varepsilon^i)\right]\left(\varepsilon^{i+1}-\varepsilon^i\right) \qquad (6)$$

Combining (4), (5) and (6) gives:

$$\left(\frac{1-\alpha_{i+1}}{\alpha_{i+1}}\right)^2 = \left(\frac{\varepsilon^i}{\varepsilon^{i+1}}\right)^2\left(\frac{1-\alpha_i}{\alpha_i}\right)^2 + \left[\frac{\sigma(\varepsilon^{i+1})+\sigma(\varepsilon^i)}{E}\right]\frac{\left(\varepsilon^{i+1}-\varepsilon^i\right)}{\left(\varepsilon^{i+1}\right)^2} \qquad (7)$$

Equation (7) reveals that as soon as the stress-strain relationship is known, the neutral axis position could easily be followed in an incremental way. Noting $k\left[\sigma(\varepsilon^{i+1})\right]$ the right term of (7), neutral axis position is directly given by (8):

$$\alpha_{i+1} = \left(1+\sqrt{k\left[\sigma(\varepsilon^{i+1})\right]}\right)^{-1} \qquad (8)$$

Main advantage of (8) is its explicit nature. Indeed, considering iterative strain loading steps, as soon as the associated stress is known (controlled by the user) the position of the neutral axis is automatically adjusted without the use of a numerical solver.

In the same way, bending moment at “i +1” iteration is obtained by integration:

$$M_{i+1} = \int_S \sigma.b.z.dz = \frac{b}{\left(\chi_{i+1}\right)^2} \times \left[\int_{\varepsilon^{i+1}}^{0} \sigma(\varepsilon).\varepsilon.d\varepsilon + \int_{0}^{\varepsilon_c^{i+1}} \sigma(\varepsilon).\varepsilon.d\varepsilon\right] = M_{i+1}^{+} + M_{i+1}^{-} \qquad (9)$$

Moment term in (9) coming from tensile stresses could easily be split incrementally:

$$M^-_{i+1}=\frac{b}{(\chi_{i+1})^2}\times\left[\int_{\varepsilon^{i+1}}^{\varepsilon^i}\sigma(\varepsilon).\varepsilon.d\varepsilon+\int_{\varepsilon^i}^{0}\sigma(\varepsilon).\varepsilon.d\varepsilon\right]=\frac{b}{(\chi_{i+1})^2}\int_{\varepsilon^{i+1}}^{\varepsilon^i}\sigma(\varepsilon).\varepsilon.d\varepsilon+\left(\frac{\chi_i}{\chi_{i+1}}\right)^2 M^-_i \quad (10)$$

The integral term has won a power compared to normal force calculation. The trapezoidal rule method is then no longer acceptable. A variant of Simpson's method is then preferred:

$$\int_{\varepsilon^{i+1}}^{\varepsilon^i}\sigma(\varepsilon).\varepsilon.d\varepsilon=\frac{\sigma(\varepsilon^{i+1}).\left(2.\varepsilon^{i+1}+\varepsilon^i\right)+\sigma(\varepsilon^i).\left(2.\varepsilon^i+\varepsilon^{i+1}\right).}{6}\times\left(\varepsilon^i-\varepsilon^{i+1}\right) \quad (11)$$

Using (3), compressive stress moment in (9) becomes :

$$M^+_{i+1}=\frac{b}{(\chi_{i+1})^2}\times\int_0^{\varepsilon_c^i}\sigma(\varepsilon).\varepsilon.d\varepsilon=\frac{E.b.\left(\varepsilon_c^{i+1}\right)^3}{3.(\chi_{i+1})^2}=-\frac{E.b}{3.(\chi_{i+1})^2}\left[\varepsilon^{i+1}\left(\frac{1-\alpha_{i+1}}{\alpha_{i+1}}\right)\right]^3 \quad (12)$$

Finally, with $\chi_{i+1}=-\varepsilon^{i+1}.\left(\alpha_{i+1}.h\right)^{-1}$, (10), (11) and (12) allow to evaluate the moment at "i+1" iteration in a incremental manner. Again, these equations are explicit as soon as the tensile constitutive equation is known, and after calculating the neutral axis position with (8).

We would expect those model equations being valid in the elastic range. This is easy to verify without approximation due to numerical integration. This result was expected for the normal force with the implementation of the trapezoidal rule to integrate elasticity linear function. By cons, it is only the implementation of the Simpson's variant method that gives an exact moment on the elasticity range (11). This choice of integration methods is one of the strengths of this modelling approach since it allows using the same set of equations to describe the whole behaviour (elastic and post-cracking).

3.2 Assessing the deflection from moment/curvature relationship

The previous model allows evaluating the intrinsic moment-curvature of a rectangular section. Experimentally, the deflection is measured and to close the loop, the link between curvature and deflection should be developed. Deflection is simply the twice integration of the curvature with the appropriate boundary conditions, neglecting shear contribution which is acceptable for thin element from a mechanical point of view. The classical elastic analytical expression for the deflection can not be used due to the non-linearity of the moment-curvature relationship (fig. 2) [10]. It is therefore necessary to further handle again with a numerical integration.

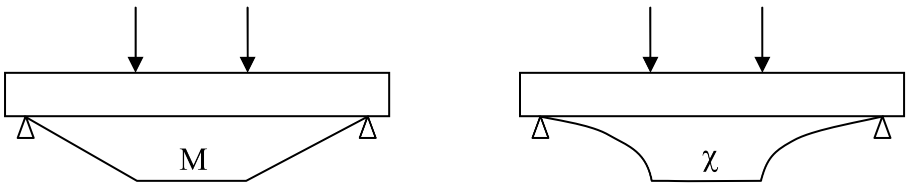

Figure 2: Schematic representation of the nonlinear distribution of the curvature

Discretization of the sample length makes the twice integration explicit and not really difficult in implementing the trapezoidal rule. It is however important to notice that in the purely elastic behaviour range, this twice numerical integration using the trapezoidal rule is not exact. Since the developed model should also allow determining the elastic modulus using the elastic range, it is then preferable to come back to the elastic equation for the deflection before matrix cracking of the matrix:

$$\delta_i = \frac{M_i}{24EI}\left(3L^2 - 4a^2\right) \tag{13}$$

Being able to compute directly the deflection, the model is now complete and can easily be implemented within a simple spreadsheet. Next section will exemplify this implantation, giving few handy tips simplifying programming and improving numerical resolution efficiency.

4. APPLYING THE MODEL FOR REVERSE ANALYSIS

The model is explicit in the direct way, meaning that starting with the stress-strain constitutive equation it allows to compute the load-deflection relationship. The strategy is to give the user the ability to adjust visually the tensile constitutive relationship helping him by a pertinent selection of key features. Let's assume that ten points are enough to describe the whole post-cracking behaviour with enough precision. At the cracking stage, the strong non-linearity behaviour makes it permissible to choose a smaller sub-division between points as illustrated by dots on fig. 3.

Even using only 10 points to describe the post-cracking behaviour, as the calculation of the deflection is not numerically exact, finer discretization is required for integration. This could be easily obtained trough loading steps. Using typically 100 loading steps, with a geometric progression of tensile strain gives satisfaction, the constitutive equation being simply linearly extrapolated for each segment (small dots on fig. 4). It is important to ensure that enough loading step are covering the elastic range to get a good precision for the numerically integrated deflection at the matrix cracking stage. We have verified that 100 leading steps with a geometric progression (until the maximum value fixed by the user) reduce the deflection error at cracking by less than one percent.

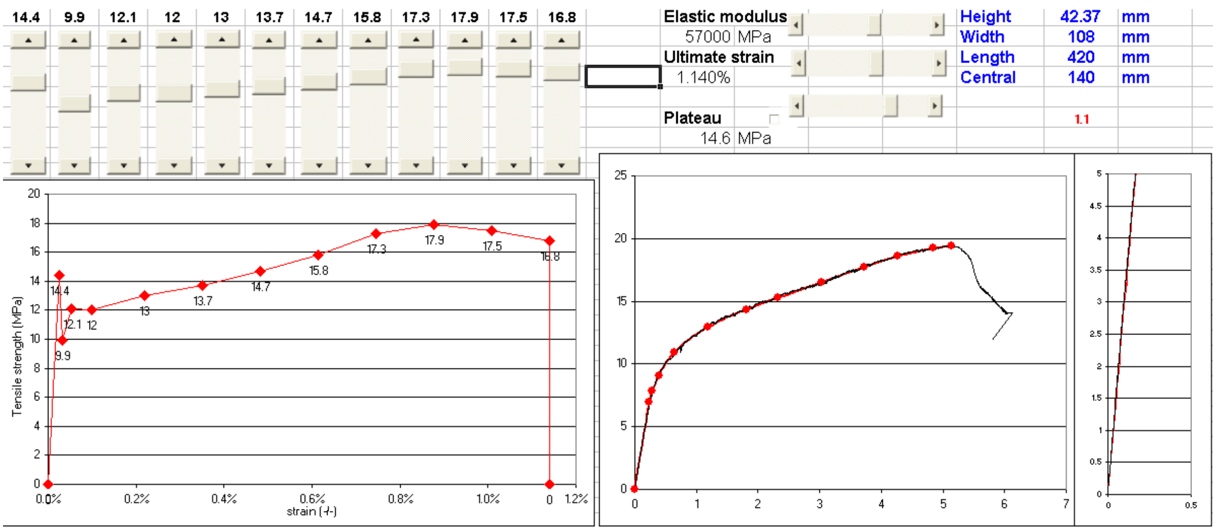

Figure 3: Screen capture of a spreadsheet allowing adjusting the tensile constitutive equation trough the cursors in order to reproduce the experimental results.

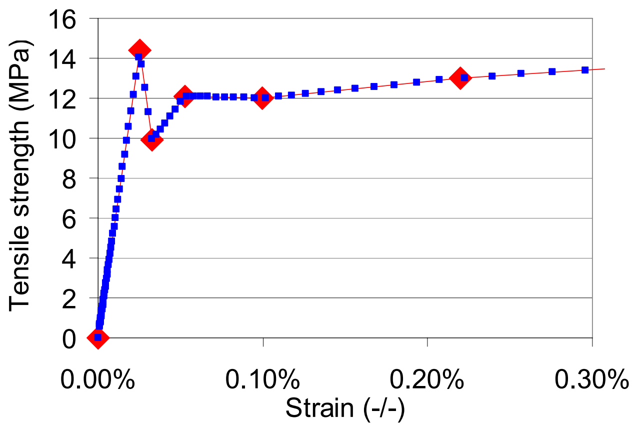

Figure 4: Geometric distribution of loading steps (small dots) against tensile behaviour (large dots).

To get the deflection trough the curvature twice integration, it is tempting to discretize the half-length of the element with a constant step. But then it requires an estimate for each abscissa of the curvature by linear interpolation of the moment-curvature relationship. It should be recognized that this additional interpolation is not necessary. Indeed, for a given loading step, all the previous loading steps provide discrete moment-curvature couples. It becomes easy to find the abscissa for those couples (trough the moment distribution over the length) and then run the twice integration. Considering the geometric progression for the loading steps, and because the moment-curvature relationship is very non linear, such dynamic way to compute the deflection gives acceptable approximation with discrete points distributed all over the element length. Another benefit of this method is that the discretization is finer as the loading is going on (more couples) which goes in the right direction for precision. As a sufficient number of loading points has been put in the elastic range to ensure precision at cracking, the numerical integration of deflection is then precise.

5. CASE STUDIES

5.1 Application to Jean Bouin Stadium

The 23,000m² envelope, including a 12,000m² roof, is made of 3600 self supporting Ductal® triangles reinforced with 2 % by volume steel fibres, each averaging 8 m to 9 m long by 2.5 m wide and 4.5 cm thick. On the façade, the panels form a light and porous web, while the roof design is simplified to permit spaces for light-diffusing glass. The slenderness of the panel allows considering that we are in the context of thin elements and previous analysis applies. To characterize mechanical properties in flexure, 4 points flexural tests on 450*110*40 mm^3 (span 420 mm) plates have bee done. First quarter on fig. 5 illustrates typical results for 6 specimens. A quite larger scatter is observed typical of a standard pouring method. Further, to keep it simple, back analysis will only be done on the four results identified with a circle, corresponding to the two best and lower performances (first quarter in fig. 5).

Second quarter on figure 5 shows that it is possible to adjust very closely the experimental curve adjusting the tensile constitutive equation. Third quarter on figure 5 illustrates the four tensile behaviours for the selected experimental results. Fibres mobilisation after cracking leads to a drop of the tensile strength but it remains enough to ensure hardening behaviour in flexure [2]. For the best specimens, ultimate tensile strength goes over the first crack strength. Finally, fourth quarter on fig. 5 illustrates the underestimation of the ultimate strain computing the deflection by using classical elastic analytical expression. In this case, the underestimation is of about 20 %!

For this case study, following the new French recommendation for UHPFRC [11], for thin slabs, the second lower ultimate strain should be adopted for design, which is around 0.8 %. The average first crack strength is 15 MPa and average E-modulus is 56 GPa.

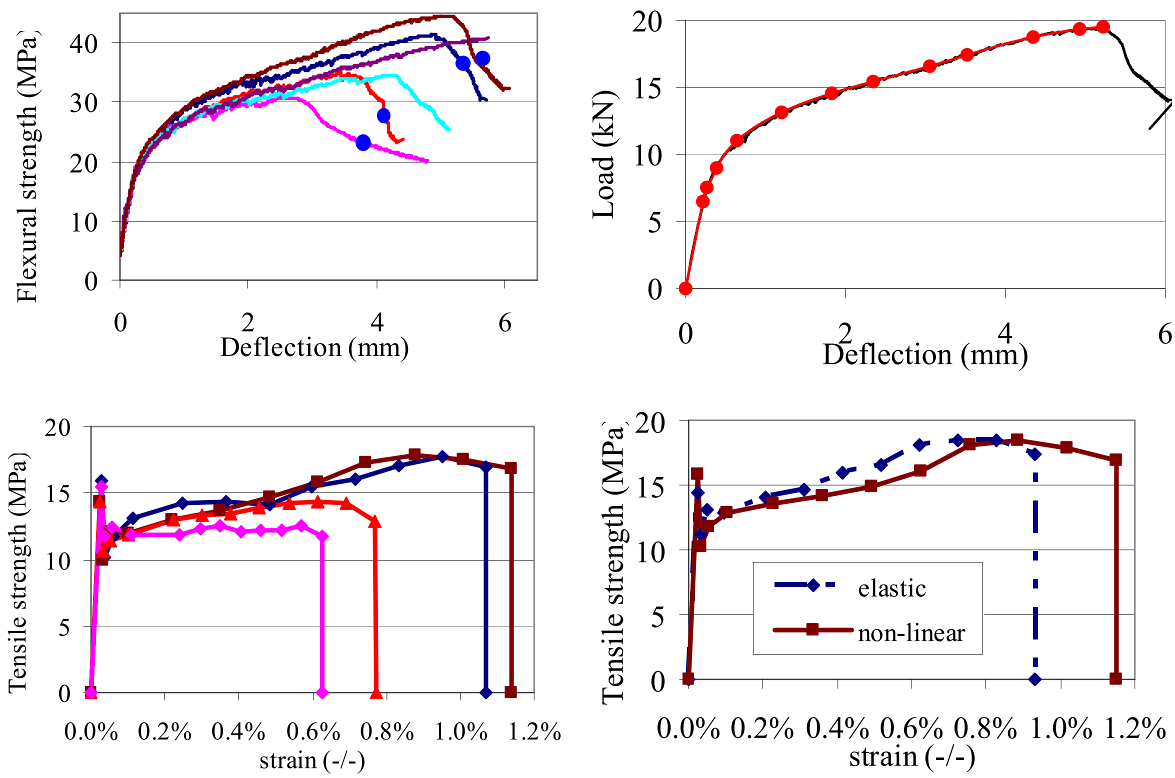

Figure 5: Back analysis on 2 vol.-% Steel fibre UHPFRC thin elements

5.2 Application to Glass Fibre UHPFRC

In 2010, we presented a paper at the UHPFRCC-4 workshop dealing with glass fibre UHPFRC [12]. At that time, back analysis of tensile constitutive equation was done adopting classical elastic equation for deflection. We go back to those results in this paragraph.

The specimens were 450 x 145 x 20 mm^3 plates with 2 % by volume of glass fibres cut from a larger plate after demoulding at 24 hours. The samples were then stored in a room at 20°C and 100 % RH for 28 days. An accelerated aging method was used to assess the long-term performance of GF-UHPRC, using the common immersion in hot water at 50°C for three additional months. Flexural results are illustrated on the left on figure 6.

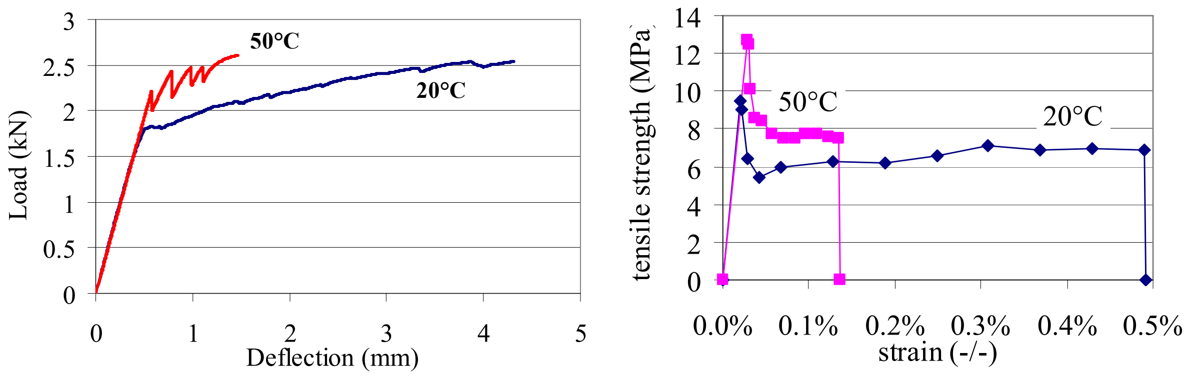

Figure 6: Back analysis on 2 % Glass fibre UHPFRC thin elements

Clearly, aging in hot water affects mechanical properties of GF-UHPFRC, as it is known for decades. Two mechanisms are in play, elastic limit increases as a result of more hydration in hot water and flexural ductility reduces. Accordingly, efficiency of the fibres should be higher to bring ductility (this is visible considering successive drops at 50°C compared to the continuous curve at 20°C on the flexural curves). Back analysis, illustrated on the right of figure 6, shows that tensile potential of this GF-UHPFRC is of the same magnitude after aging compared to reference even slightly higher but the amplitude of ultimate strain is reduced, which explains reduced flexural ductility.

Whatever such aging effect, well known with glass fibres, it is at the structural level, meaning depending on element size, that ductility should be address for thin elements. Indeed, as illustrated on figure 6, mechanical potential remains unaffected by aging with hardening post-cracking mechanical potential.

6. CONCLUSION

Thin element made of fibre reinforced concrete having a hardening behaviour under flexure could be analysing in term of stress/strain constitutive equation up to the maximum post-cracking moment. A reverse analysis method could then be used to extract the material tensile constitutive equation from flexural tests. The objective of such reverse analysis is not to give exact description of the fibres contribution but to approximate the mechanical properties for further element design. It is always tempting to develop sophisticated models and software to run such back analysis. However, it should be kept in mind that at the end, the tensile constitutive equation is generally approximate as an elastic-plastic model for element design purpose.

We developed in this paper a set of equation to describe the mechanical behaviour of thin elements under flexure. Expressing those equations into an incremental form allows solving them (from constitutive equations to flexural behaviour) without the need of any solver, even considering non-linear post-cracking behaviour. It falls then to the user to adjust the constitutive equation superposing visually the theoretical curve and the experimental one. This provides two main advantages: i) the computation is very fast, ii) visual superposition allows removing de facto experimental noises.

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