THEORETICAL MODEL FOR SIZE AND SHAPE EFFECT OF UHPFRC IN FLEXURAL TENSION CONSIDERING TENSILE BEHAVIOUR INFLUENCED BY FIBRE ORIENTATION

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Abstract

The fibre orientation in UHPFRC members influences significantly the structural behaviour and strength and needs to be considered in design. The stress – crack opening law could be described by a fibre model and this corresponded well to own tensile tests.

Between the fibre orientation and the tensile strength a nonlinear function was derived. The stress – crack opening law based on the fibre model for a one dimensional fibre orientation was approximated by a piece-wise function, and this was used to describe the constitutive law in tension of UHPFRC considering the fibre orientation.

To explain the experimentally verified size and shape effect of the flexural tensile strength, an original fracture mechanics model was developed following the "fictitious crack model" of Hillerborg et al. (1976) and compared to other models. It was found that fracture mechanics alone cannot explain the size and shape effect of the flexural strength of UHPFRC but a theoretical explanation is only possible by additionally considering the fibre orientation.

Résumé

L'orientation des fibres dans les structures en BFUP a une influence importante sur leur comportement et leur résistance et doit être prise en compte dans le calcul. La loi contrainte-ouverture de fissure a pu être décrite par un modèle associé à la contribution des fibres et correspond bien au résultat d'essais de traction.

Une relation non-linéaire a été identifiée entre l'orientation des fibres et la résistance en traction. La loi contrainte-ouverture de fissure basée sur la modélisation des fibres pour une orientation unidirectionnelle de celles-ci a été approchée par une fonction comprenant plusieurs stades, et c'est cette fonction qui a été prise en compte pour établir la loi de comportement en traction du BFUP en intégrant l'effet d'orientation des fibres.

Pour expliquer l'effet d'échelle et l'effet de structure, avérés expérimentalement, sur la résistance en traction par flexion, un modèle original basé sur la mécanique de la rupture a été mis au point en s'inspirant du "modèle de la fissure fictive" de Hillerborg et al. (1976) et a été comparé à d'autres approches. On trouve que la mécanique de la rupture à elle seule ne suffit pas à expliquer l'effet d'échelle et l'effet de structure observés sur la résistance en flexion du BFUP, et qu'une explication théorique n'est possible qu'en tenant également compte de l'orientation des fibres.

1. OPTICAL FIBRE ORIENTATION MEASUREMENT

A basic requirement for the optical measurement of the fibre orientation are high quality "low key" digital pictures of the specimen cross sections which allow a clear distinction between the matrix and the steel fibres. Such a measurement set-up was designed by Frettlöhr (2011) and successfully applied. The principal diameters of each fibre and the number of fibres are determined by applying an algorithm derived by Frettlöhr (2011) using the method of optical measurement with digital image processing. The algorithm was programmed using the HALCON 8 machine vision development environment by MVTec. For further details see Frettlöhr (2011).

2. THEORETICAL CONSIDERATIONS REGARDING TENSILE STRENGTH

2.1 Fibre orientation and correlation to tensile strength

The uniaxial tensile strength of UHPFRC depends on the fibre orientation and this was shown indirectly with bending tests of BPR specimens by Bernier and Behloul (1996). A special formwork following a proposal by Hannant et al. (1974) was used for casting a plate in order to ensure a nearly one dimensional alignment of the fibres into the flow direction. Test specimens were cut out of the platter under different angles to achieve a certain fibre orientation and were tested in bending. The flexural tensile strength clearly decreased with an increasing angle between the fibres and the principle flexural stress direction.

Based on the constitutive law of BPR in compression and tension by Behloul (1996a, 1996b) the required uniaxial tensile strength $f_{ct}(\eta)$ was determined by Frettlöhr (2011). It is related to the fibre orientation of each specimen in order to achieve the flexural tensile strength $f_{ct,fl}$ of the test specimens. The one dimensional tensile strength $f_{ct,1D}$ is achieved when all fibres are orientated into the principle stress direction and it is the upper limit of the uniaxial tensile strength, which is 18.4 MPa for BPR according to Behloul (1996b). For the bending test with a one dimensional fibre orientation Frettlöhr (2011) calculated a value of $f_{ct,1D}$ = 17.8 MPa which fits very well to the value by Behloul (1996b). In Fig. 1 the ratio $f_{ct}(\eta)/f_{ct,1D}$ versus the fibre orientation η is plotted as derived from the bending tests by Bernier and Behloul (1996), which characterized samples of varying (and uniform) fibre orientation. The correlation between the ratio $f_{ct}(\eta)/f_{ct,1D}$ and the fibre orientation η is clearly nonlinear. This contradicts the so far assumed linear correlation (shown as black dashed line in Fig. 1) of several other publications, e.g. Behloul (1996b) and Leutbecher (2007).

The influence of the fibre orientation on the tensile strength of SIFCON was investigated by van Mier and Timmers (1991) and the nonlinear correlation is confirmed (Fig. 1). The curve for SIFCON deviates from that for BPR probably due to the far higher fibre content of circa 11,5 Vol.-% compared to circa 2 Vol.-% and the two times longer fibres. Furthermore also the higher fibre diameter of SIFCON might be a reason. One would expect for a fibre orientation of η = 0 (all fibre are orientated perpendicular to the principle stress direction) a ratio $f_{ct}(\eta)/f_{ct,1D}$ of 0 for BPR. Therefore a corrected trend for the correlation is proposed in Fig. 4. The proposed correlation between the tensile strength and the fibre orientation can be described well by the following equation shown in Fig. 2:

$$\chi(\eta) = \frac{f_{ct}(\eta)}{f_{ct,1D}} = 0.0014 \cdot e^{(5.97 \cdot \eta + 0.22)} + 0.32 \cdot \eta \qquad (1)$$

where: $\chi(\eta)$ ratio of fibre orientation related tensile strength $f_{ct}(\eta)$ to one dimensional tensile strength $f_{ct,1D}$

It should be pointed out that a ratio $f_{ct}(\eta)/f_{ct,1D}$ of 0 does not mean that the matrix strength, which is equal to the tensile stress at the elastic limit $\sigma_{ct,el}$, is also 0. Rather the fibres do not improve the tensile strength after exceeding the elastic limit.

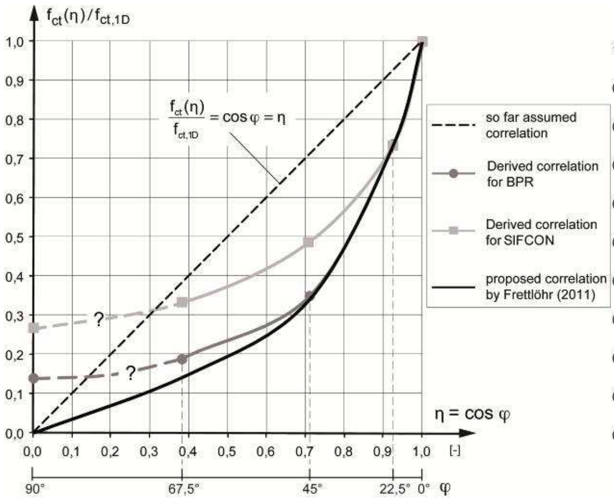


Figure 1: Ratio $f_{ct}(\eta)/f_{ct,1D}$ versus fibre orientation factor η

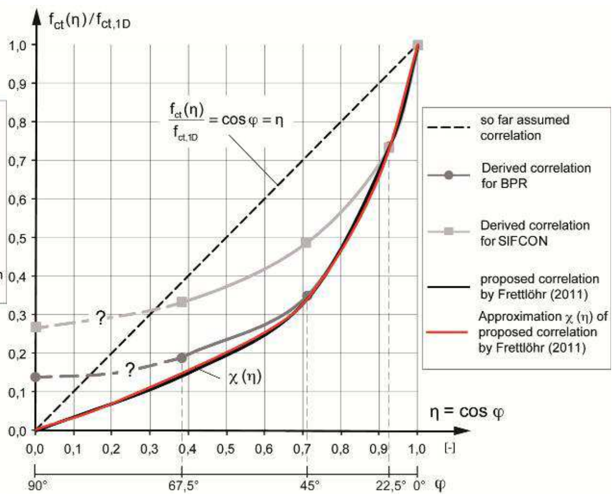


Figure 2: Proposed approximation by Frettlöhr (2011) for ratio $f_{ct}(\eta)/f_{ct,1D}$

2.2 Stress – crack opening law based on fibre model

The bond behaviour of concrete and steel reinforcement can be described through a differential equation (DE) by Kuuskoski (1950) and Rehm (1961). This DE applies also for fibre reinforced concrete and the stress-crack opening law of a single fibre can be derived. In order to solve the DE a bond law has to be assumed. In Fig. 3 three different bond laws are shown proposed by Pfyl (2003), Namur et al. (1989) and Naaman et al. (1991a). The bond law proposed by Namur et al. (1989) is a special case of that from Naaman et al. (1991a).

The DE was solved analytically by Frettlöhr (2011) considering the bond law by Pfyl (2003) and Naaman et al. (1991a). He furthermore developed a numerical algorithm based on the DE solution for a single fibre to determine the stress-crack opening law and the one dimensional tensile strength $f_{ct,1D}$ of an arbitrary cross section.

A parameter study by Frettlöhr (2011) revealed that neither the assumed bond distribution nor its absolute value had an influence on the 1D tensile strength for a one dimensional fibre orientation (the upper limit of the uniaxial tensile strength) and the curve of the descending branch of the stress – crack opening law. The bond only results in a steeper ascending branch of the stress – crack opening law. For his further investigations Frettlöhr (2011) applied therefore the bond law by Pfyl (2003).

The derived normalized stress – crack opening law corresponded well to the measured crack opening behaviour of own tensile tests with unnotched specimens (Fig. 4). Also a comparison was done with the proposals by Li (1992), Naaman et al. (1993) and Behloul et al. (1996) (Fig. 4) and showed that the derived stress – crack opening law by Frettlöhr (2011) exhibited the best agreement with the test curves of specimens out of Ductal®.

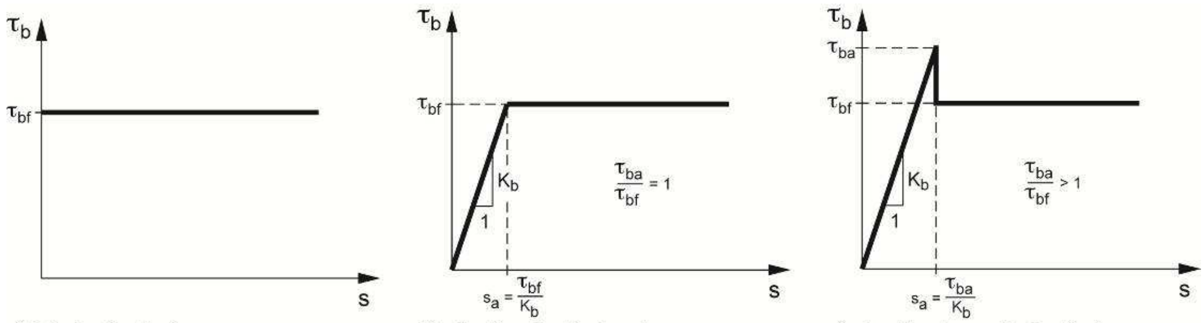


Figure 3: Bond laws

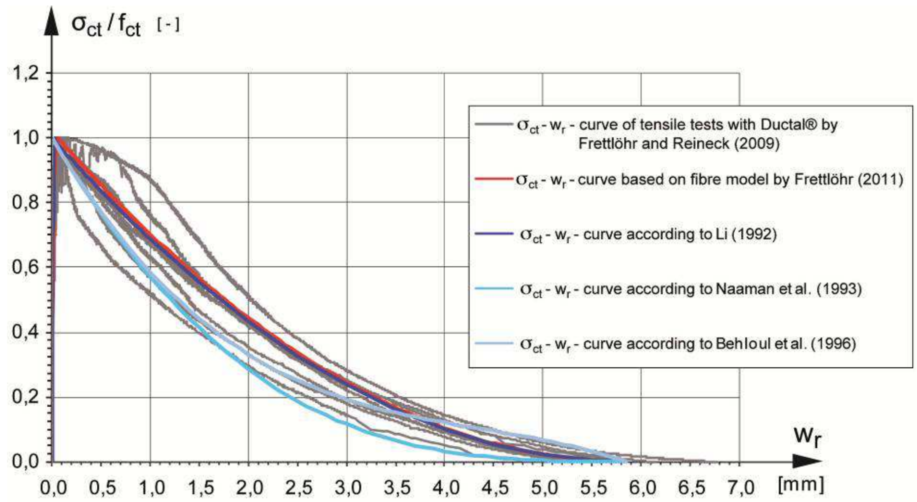


Figure 4: Different theoretical stress – crack opening laws compared to tensile tests with Ductal® by Frettlöhr and Reineck (2009)

2.3 Constitutive law in tension of UHPFRC

The numerically derived descending branch of the stress – crack opening law is appropriately approximated by the following equation.

$$\frac{\sigma_{ct}(w_r)}{f_{ct,1D}} = \left(1 - \frac{2 \cdot (w_r - w_r(f_{ct,1D}))}{l_f}\right)^2 \quad for\ w_r \geq w_r\left(f_{ct,1D}\right) \tag{2}$$

where: w_r [mm] crack opening; crack width
$w_r(f_{ct,1D})$ [mm] crack opening corresponding to $f_{ct,1D}$
$f_{ct,1D}$ [MPa] tensile strength for one dimensional fibre orientation
l_f [mm] fibre length

The ascending branch is approximated by a simplified linear curve:

$$\frac{\sigma_{ct}(w_r)}{f_{ct,1D}} = \left(1 - \frac{\sigma_{ct,el}}{f_{ct,1D}}\right) \cdot \frac{w_r}{w_r(f_{ct,1D})} + \frac{\sigma_{ct,el}}{f_{ct,1D}} \quad \text{für } w_r < w_r\left(f_{ct,1D}\right) \tag{3}$$

where: $\sigma_{ct,el}$ [MPa] tensile stress at the elastic limit

In order to define a constitutive law in tension of UHPFRC the Eqs. (2) and (3) have to be extended to consider the influence of the fibre orientation as described by Eq. (1). Reference is made to Frettlöhr (2011) for the mathematical equations of the constitutive law in tension including the fibre orientation. In Fig. 5 the principle constitutive law in tension is shown for different fibre orientation factors; please note that the ascending branch is not to scale. For the very high values of η = 1 and η = 0.9 of the fibre orientation the tensile strength f_{ct} is higher than $\sigma_{ct,el}$, whereas it is lower for values than about η = 0.5.

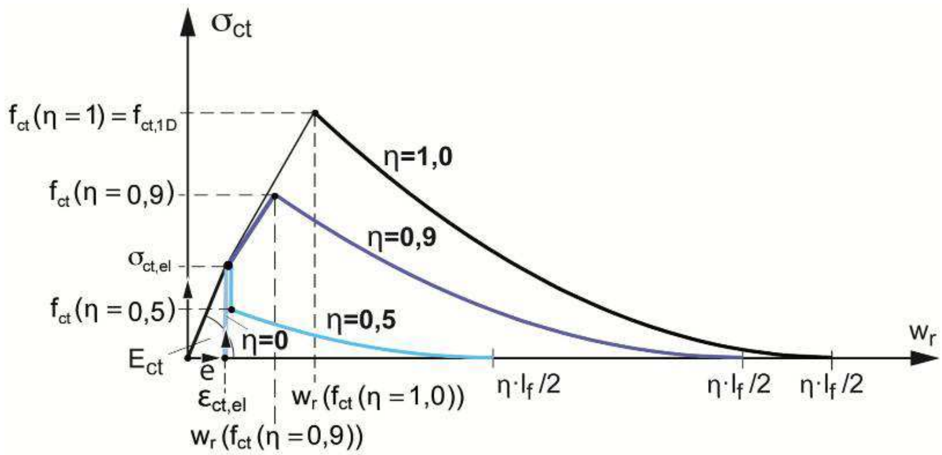


Figure 5: Constitutive law in tension of UHPFRC for different fibre orientation factors η (ascending branch is not to scale)

3. THEORETICAL MODEL FOR SIZE AND SHAPE EFFECT

3.1 Fracture Mechanics Model

A fracture mechanics model was developed by Frettlöhr (2011) following the „fictitious crack model" of Hillerborg et al. (1976) to explain the experimentally verified size and shape effect of the flexural tensile strength. In general calculations according to the "fictitious crack model" are performed by use of finite element analysis with discrete crack modelling. To avoid this, the own model is based on a cross section consideration. The challenge hereby is that the load bearing behaviour of the compression zone is described by a stress – strain relationship whereas in the tensile zone a combination of a stress – strain relationship for the elastic part and a stress – crack opening law has to be applied. Therefore a compatibility condition has to be defined in order to link strains to crack opening. An overview of different proposals of kinematic approaches for the compatibility condition by Pederson, Casanova and Rossi as well as Olesen is given by RILEM TC 162-TDF (2002).

Casanova and Rossi (1996) use the model shown in Fig. 6 to define their compatibility condition. The beam is divided into an uncracked cross section following the elasticity theory and a rigid body in the cracked area along the crack influence length. The influence length of the crack is considered as twice the crack length. Since the curvature course χ is unknown within the crack influence length so it is assumed to be parabolic by Casanova and Rossi (1996).

To avoid this assumption, for the own fracture model an alternative compatibility condition is defined which links the crack opening w_r with the elongation along the plastic hinge in the compression zone of the cracked cross section (Fig. 8). Following the strut and tie model theory the crack area is considered as a D-Region and therefore the crack influence length is twice the beam height h according to Schlaich and Schäfer (2001). This differs from the assumption of twice the crack length by e.g. Casanova and Rossi (1996) as well as Fehling and Leutbecher (2011). The beam area within the crack influence length is modelled analogue to Casanova and Rossi (1996) (Fig. 12a).

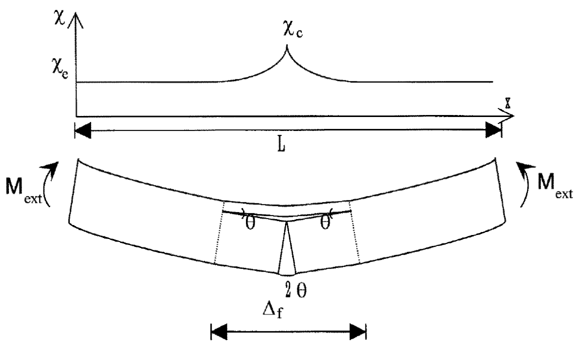


Figure 6: Kinematic approach by Casanova and Rossi (1996)

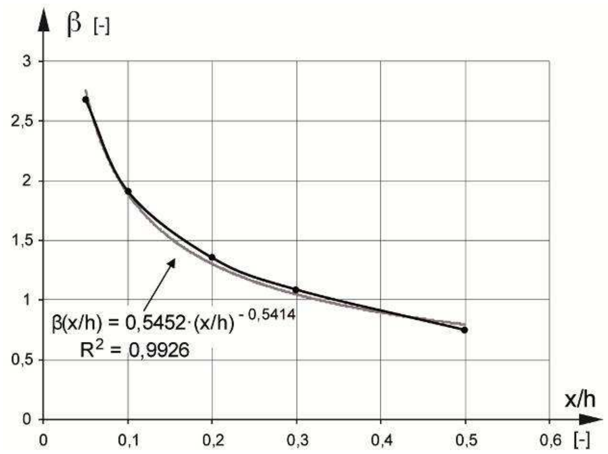


Fig. 7: Course of run-out factor β as a function of x/h

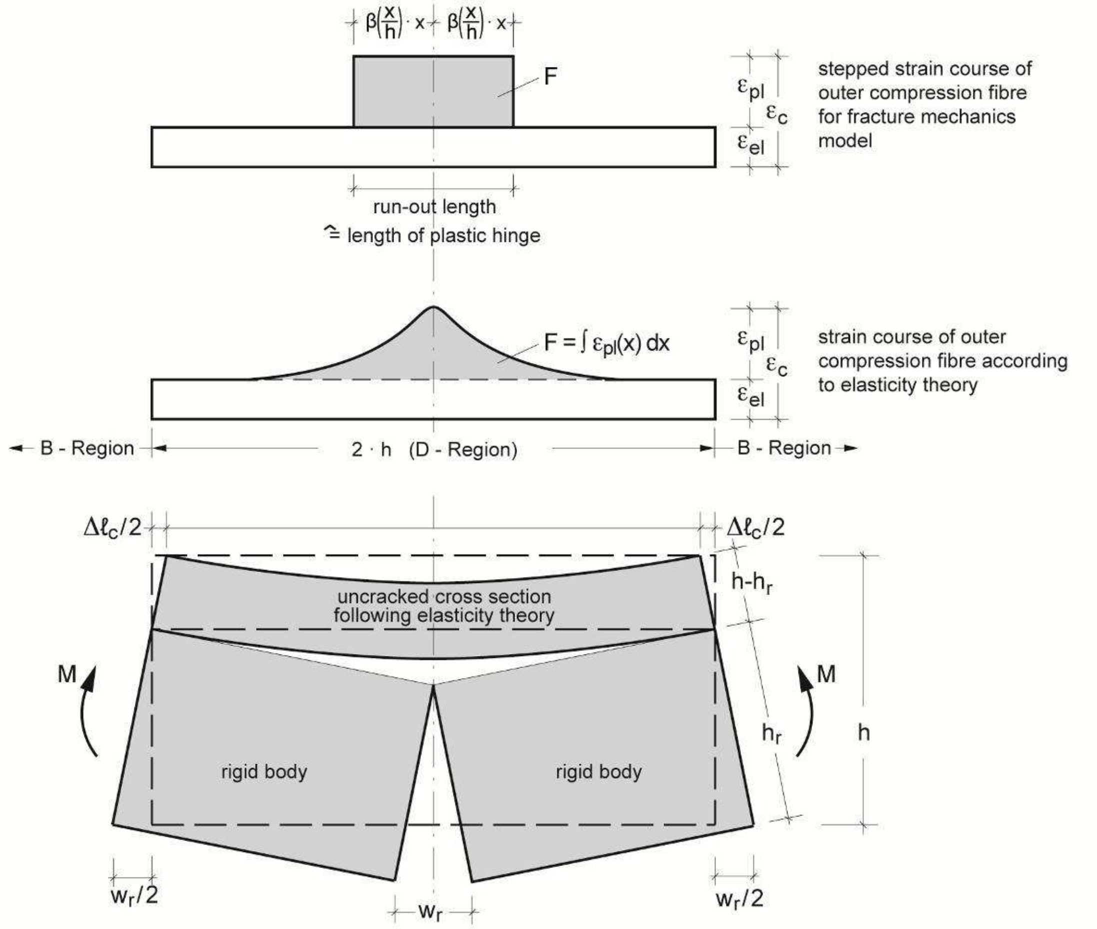
The strain course of the outer compression fibre $\varepsilon_c(x)$ is derived by linear elastic finite element calculations for varying compression zone heights x and can be divided into an elastic and plastic part (Fig. 8a). Integration of the plastic strain course leads to a stepped strain course (Fig. 8a) with a run-out length of $2 \cdot \beta(x/h) - x$ which is equal to the plastic hinge length. The course of the run-out factor β as a function of b/h and the applied approximation equation for the fracture mechanics model is shown in Fig. 7.

The overall elongation Δl_c consists of an elastic and plastic part. Due to symmetry only the half system is considered in the following. At the transition from the D- to the B-Region the Bernoulli-Hypotheses is valid and the distribution of the elongation $\Delta l(z)$ along the cross section height is shown in Fig 8 b1. Due to the rigid body movement the elongation at the lower edge of the tensile zone is $\Delta l(z = 0) = w_r/2$ and at the upper egde of the compression zone $\Delta l(z = h) = \Delta l_c(z = h)/2$.

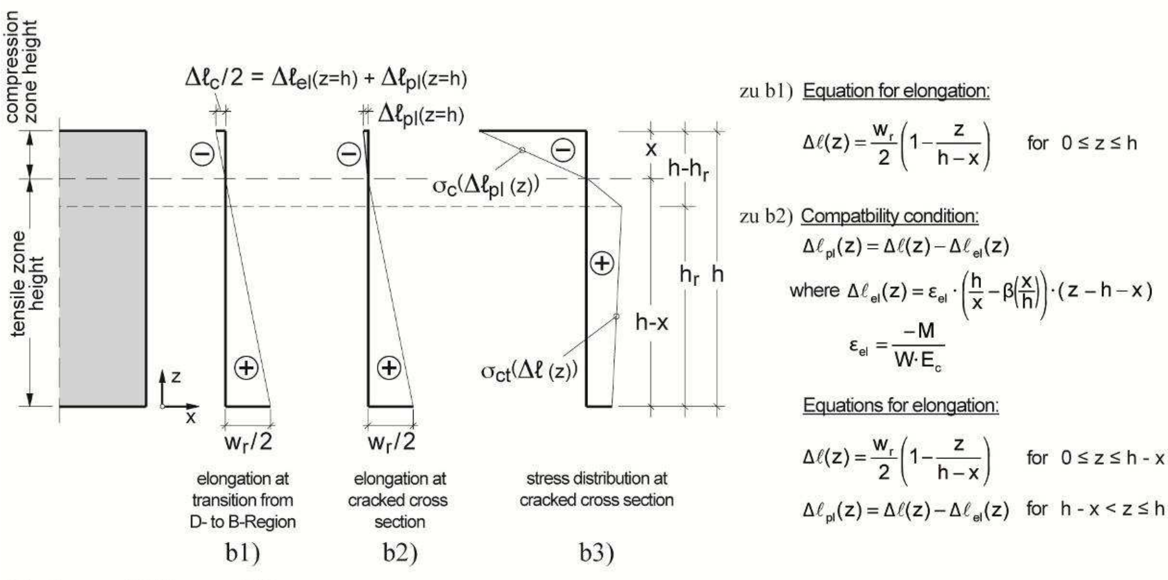
The elongation of the compression zone consists of a elastic part $\Delta l_{el}(z)$ and a plastic part $\Delta l_{pl}(z)$:

$$\Delta l(z) = \Delta l_{el}(z) + \Delta l_{pl}(z) \tag{4}$$

The elastic elongation $\Delta l_{el}(z)$ is derived (Fig. 8 b2) by integrating of the elastic strain ε_{el} along the D-Region length less the plastic hinge length. Rewriting of Eq. (4) gives the plastic elongation $\Delta l_{pl}(z)$ and therefore the compatibility condition (Fig. 8 b2).



a) Fracture mechanics model for determination of flexural tensile strength f_{ctfl}



b) Compatibility condition

Figure 8: Fracture model for size and shape effect of flexural tensile strength
by Frettlöhr (2011)

It has to be pointed out that within the cracked cross section the Bernoulli-Hypothesis does not apply. This is expressed by a buckle of the elongation course along the height at the neutral axis. The equations for describing the elongation along the cross section height are given in Fig. 8 b2. For further details and a calculation algorithm for the proposed fracture mechanics model see Frettlöhr (2011).

3.2 Comparison with other models and tests

The flexural tensile strength of test series by Frettlöhr and Reineck (2009) as well as Reineck and Frettlöhr (2011) was calculated using the fracture mechanics model of chapter 3.1 in combination with the proposed constitutive law considering the influence of the fibre orientation (Fig. 5). The fibre orientation was measured by newly designed measurement set-up as described in section 1. The results were compared with the tests and the following proposals:

- Model by Casanova and Rossi (1996);
- AFGC/SETRA (2002) recommendation for transforming the stress - crack opening relationship into a stress-strain relation by applying a “structural length“ of $l_{stl} = 2/3 \cdot h$;
- “Structural length“ of $l_{stl} = 2 \cdot (h-x)$ proposed by Fehling and Leutbecher (2011) for transforming the stress - crack opening relationship into a stress-strain relationship.

In Fig. 9 the flexural tensile strength versus the height h for b/h = 3 are compared for the different approaches. The flexural tensile strength of the test series decreases by ~35 % with an increasing specimen height from h = 25 to 150 mm. The approaches by Casanova and Rossi (1996), AFGC/SETRA (2002) and Fehling and Leutbecher (2011) each show a decrease of the flexural tensile strength of only about 12 to 16 % and match the test results for h = 75 respectively 100 mm. However, the model proposed by Frettlöhr (2011) exhibits smaller discrepancies in comparison to the test results with differences of only 3 to 7 %. The theoretical flexural tensile strength for h = 50 mm reaches only 81 % of the test value, but this test appears to be an outlier. A comparison of the test series for b/h = 1 and 5 show similar results as described for b/h =3. The model by Frettlöhr (2011) explains also well the shape effect as derived by tests.

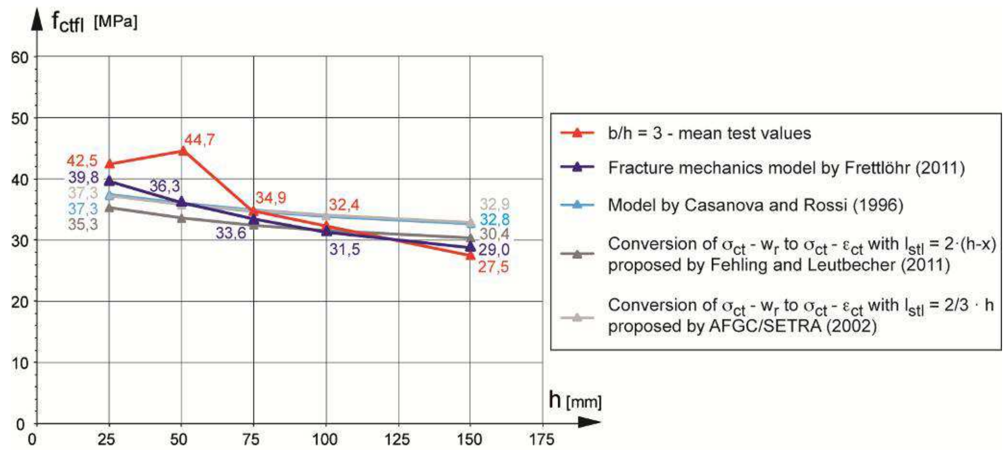


Figure 9: Test results for flexural tensile strength $f_{ct,fl}$ versus height h compared with the different approaches

The model by Casanova and Rossi (1996) as well as the own model are based on fracture mechanics. In contrast, both proposals for a "structural length" for transforming the stress - crack opening relationship into a stress - strain relationship by AFGC/SETRA (2002) and Fehling and Leutbecher (2011) are based on a smeared numerical approach. The compression zone is considered as a strut with constant strain. Therefore these models neglect the local strain and stress increase within the compression zone when the localisation occurs. Reference is made to Frettlöhr (2011) for further details.

4. CONCLUSIONS

It was shown that the correlation between the fibre orientation and the tensile strength is nonlinear. Furthermore the stress – crack opening law of a single fibre could be described by a fibre model. A numerical algorithm based on the fibre model enabled to determine the stress-crack opening law of a arbitrary cross section and to estimate the one dimensional tensile strength $f_{ct,1D}$. For UHPFRC in tension a constitutive law was defined considering the influence of the fibre orientation. The proposed fracture mechanics model in combination with the constitutive law in tension of UHPFRC considering the influence of the fibre orientation give a theoretical explanation for the experimentally determined size and shape influence on the flexural tensile strength.

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